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# Electroweak and supersymmetric two-loop corrections to lepton anomalous magnetic and electric dipole moments

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## Abstract

Using the effective Lagrangian method, we analyze the electroweak corrections to the anomalous dipole moments of lepton from some special two-loop diagrams where a closed neutralino/chargino loop is inserted into relevant two Higgs doublet one-loop diagrams in the minimal supersymmetric extension of the standard model with CP violation. Considering the translational invariance of loop momenta and the electromagnetic gauge invariance, we get all dimension 6 operators and derive their coefficients. After applying equations of motion to the external leptons, we obtain the anomalous dipole moments of lepton. The numerical results imply that there is parameter space where the contributions to the muon anomalous dipole moments from this sector may be significant.

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## 1. Introduction

At both aspects of experiment and theory, the magnetic dipole moments (MDMs) of lepton draw the great attention of physicists because of their obvious importance. The anomalous dipole moments of lepton not only can be used for testing loop effect in the standard model (SM), but also provide a potential window to detect new physics beyond the SM. The current experimental

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result of the muon MDM is [1,2]

$$a_{\mu}^{\text{exp}} = 11\,659\,208 \pm 6 \times 10^{-10}. \quad (1)$$

From the theoretical point of view, contributions to the muon MDM are generally divided into three sectors [2,3]: QED loops, hadronic contributions and electroweak corrections. The largest uncertainty of the SM prediction originates from the evaluation of hadronic vacuum polarization and light-by-light corrections. Depending on which evaluation of hadronic vacuum polarization is chosen, the differences between the SM predictions and experimental result are given as [2,3]:

$$\begin{aligned} a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} &= 33.2 \pm 8.8 \times 10^{-10}: \quad 3.8\sigma, \\ a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} &= 30.5 \pm 9.3 \times 10^{-10}: \quad 3.3\sigma, \\ a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} &= 28.2 \pm 8.9 \times 10^{-10}: \quad 3.2\sigma, \\ a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} &= 11.9 \pm 9.5 \times 10^{-10}: \quad 1.3\sigma. \end{aligned} \quad (2)$$

For the convenience of numerical discussion, we will adopt the second value in Eq. (2). Within three standard error deviations, this difference implies that the present experimental data can tolerate new physics correction to the muon MDM as

$$2.6 \times 10^{-10} \leq \Delta a_{\mu}^{\text{NP}} \leq 58.4 \times 10^{-10}. \quad (3)$$

In fact, the current experimental precision ( $6 \times 10^{-10}$ ) already puts very restrictive bounds on new physics scenarios. In the SM, the electroweak one- and two-loop contributions amount to  $19.5 \times 10^{-10}$  and  $-4.4 \times 10^{-10}$ , respectively. Comparing with the standard electroweak corrections, the electroweak corrections from new physics are generally suppressed by  $\Lambda_{\text{EW}}^2/\Lambda^2$ , where  $\Lambda_{\text{EW}}$  denotes the electroweak energy scale and  $\Lambda$  denotes the energy scale of new physics.

Supersymmetry (SUSY) has been considered as a most prospective candidate for new physics beyond the SM. In the minimal supersymmetric extension of the SM (MSSM) with CP conservation, the supersymmetric one-loop contribution is approximately given by

$$\Delta a_{\mu}^{1L} \simeq 13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{\Lambda} \right)^2 \tan \beta \text{sign}(\mu_H), \quad (4)$$

when all supersymmetric masses are assumed to equal a common mass  $\Lambda$ , and  $\tan \beta = \frac{v_2}{v_1} \gg 1$ . Where  $v_1$  and  $v_2$  are the absolute values of the vacuum expectation values (VEVs) of the Higgs doublets and  $\mu_H$  denotes the  $\mu$ -parameter in the superpotential of MSSM. It is obvious that the supersymmetric effects can easily account for the deviation between the SM prediction and the experimental data.

Actually, the two-loop electroweak corrections to the anomalous dipole moments of lepton are discussed extensively in literature. Utilizing the heavy mass expansion approximation (HME) together with the corresponding projection operator method, Ref. [4] has obtained the two-loop standard electroweak correction to the muon MDM which eliminates some of the large logarithms that were incorrectly kept in a previous calculation [5]. Within the framework of MSSM with CP conservation, the authors of Refs. [6,7] present the supersymmetric corrections from some special two-loop diagrams where a close chargino (neutralino) loop or a scalar fermion loop is inserted into those two-Higgs-doublet one-loop diagrams. Ref. [8] discusses the contributions to the muon MDM from the effective vertices  $H^{\pm}W^{\mp}\gamma$ ,  $h_0(H_0)\gamma\gamma$  which are induced by the scalar quarks of the third generation in the CP conserving MSSM.

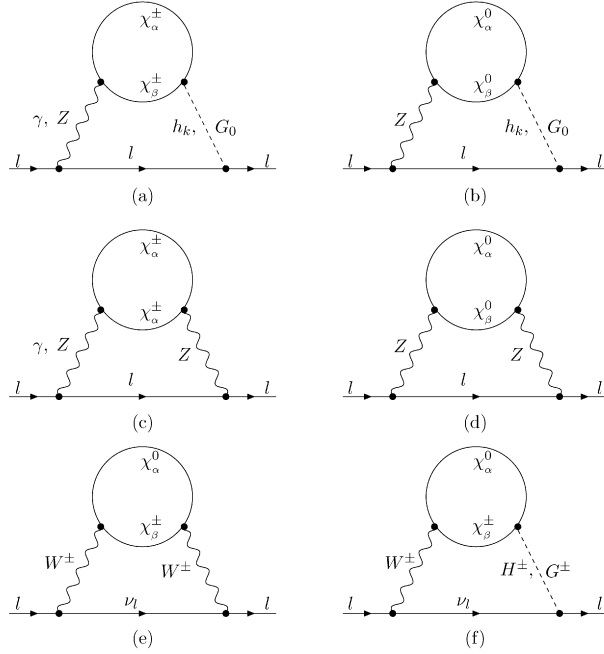


Fig. 1. Some two-loop self energy diagrams which lead to the lepton MDMs and EDMs in CP violating MSSM, the corresponding triangle diagrams are obtained by attaching a photon in all possible ways to the internal particles. In concrete calculation, the contributions from those mirror diagrams should be included also.

In this paper, we investigate the electroweak corrections to the anomalous dipole moments of lepton from some special two-loop diagrams where a closed neutralino/chargino loop is inserted into relevant two Higgs doublet one-loop diagrams in the CP violating MSSM (Fig. 1). Since the masses of those virtual fields ( $W^\pm$ ,  $Z$  gauge bosons, neutral and charged Higgs, as well as neutralinos and charginos) are much heavier than the muon mass  $m_\mu$ , we can apply the effective Lagrangian method to get the anomalous dipole moments of lepton. After integrating out the heavy freedoms mentioned above and then matching between the effective theory and the full theory, we derive the relevant higher dimension operators as well as the corresponding Wilson coefficients. The effective Lagrangian method has been adopted to calculate the two-loop supersymmetric corrections to the branching ratio of  $b \rightarrow s\gamma$  [9], neutron EDM [10] and lepton MDMs and EDMs [11]. In concrete calculation, we assume that all external leptons as well as photon are off-shell, then expand the amplitude of corresponding triangle diagrams according to the external momenta of leptons and photon. Using loop momentum translational invariance, we formulate the sum of amplitude from those triangle diagrams which correspond to the corresponding self-energy in the form which explicitly satisfies the Ward identity required by the QED gauge symmetry. Then we can get all dimension 6 operators together with their coefficients. After the equations of motion are applied to external leptons, higher dimensional operators, such as dimension 8 operators, also contribute to the muon MDM and the electron EDM in principle. However, the contributions of dimension 8 operators contain an additional suppression factor  $m_l^2/\Lambda^2$  comparing with that of dimension 6 operators, where  $m_l$  is the mass of lepton. Setting  $\Lambda \sim 100$  GeV, one obtains easily that this suppression factor is about  $10^{-6}$  for the muon lepton.

Under current experimental precision, it implies that the contributions of all higher dimension operators ( $D \geq 8$ ) can be neglected safely.

We adopt the naive dimensional regularization with the anticommuting  $\gamma_5$  scheme, where there is no distinction between the first 4 dimensions and the remaining  $D - 4$  dimensions. Since the bare effective Lagrangian contains the ultraviolet divergence which is induced by divergent subdiagrams, we give the renormalized results in the on-mass-shell scheme [12]. Additional, we adopt the nonlinear  $R_\xi$  gauge with  $\xi = 1$  for simplification [13]. This special gauge-fixing term guarantees explicit electromagnetic gauge invariance throughout the calculation, not just at the end because the choice of gauge-fixing term eliminates the  $\gamma W^\pm G^\mp$  vertex in the Lagrangian.

Within the framework of CP violating MSSM, the renormalization-group improved loop effects of soft CP violating Yukawa interactions related to scalar quarks of the third generation cause the strong mixing among CP-even and CP-odd neutral Higgs. The linear expansions of the Higgs doublet  $H^1$  and  $H^2$  around the ground state are generally written as

$$H^1 = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_1 + \phi_1^0 + ia_1) \\ \phi_1^- \end{pmatrix}, \quad H^2 = e^{i\theta} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2^0 + ia_2) \end{pmatrix}, \quad (5)$$

where  $\theta$  is their relative phase. In the weak basis  $\{\phi_1^0, \phi_2^0, a = \sin\beta a_1 + \cos\beta a_2\}$ , the neutral mass-squared matrix  $M_H^2$  may be expressed as

$$M_H^2 = \begin{pmatrix} (\mathcal{M}_S^2)_{11} & (\mathcal{M}_S^2)_{12} & \frac{1}{\cos\beta}(\mathcal{M}_{SP}^2)_{12} \\ (\mathcal{M}_S^2)_{12} & (\mathcal{M}_S^2)_{22} & -\frac{1}{\sin\beta}(\mathcal{M}_{SP}^2)_{21} \\ \frac{1}{\cos\beta}(\mathcal{M}_{SP}^2)_{12} & -\frac{1}{\sin\beta}(\mathcal{M}_{SP}^2)_{21} & -\frac{1}{\sin\beta\cos\beta}(\mathcal{M}_{SP}^2)_{12} \end{pmatrix}. \quad (6)$$

Here, the concrete expressions of  $(\mathcal{M}_S^2)_{ij}$ ,  $(\mathcal{M}_{SP}^2)_{ij}$  can be found in the literature [14]. Since the Higgs mass matrix  $M_H^2$  is symmetric, we can diagonalize it by an orthogonal rotation  $Z_H$  as:

$$Z_H^T M_H^2 Z_H = \text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2). \quad (7)$$

Because of this strong mixing among the neutral Higgs, the couplings involving neutral Higgs are modified drastically comparing with that in CP conserving MSSM. Certainly, some diagrams in Fig. 1 have been discussed in Ref. [7] where the authors apply the projecting operators to get the lepton MDMs (Eqs. (8)–(10) in Ref. [7]) within the framework of CP conserving MSSM. On the other hand, the fermion electric dipole moments (EDMs) also offer a powerful probe for new physics beyond the Standard Model (SM). In the SM, the EDMs of leptons are fully induced by the CP phase of the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements and they are predicted to be much smaller [15] than the present experimental precision [16,17] and beyond the reach of experiments in the near future. As for the MSSM, there are many new sources of the CP violation that can result in larger contributions to the EDMs of electron and neutron [18,19]. Taking the CP phases with a natural size of  $\mathcal{O}(1)$ , and the supersymmetry mass spectra at the TeV range, we can find that the theoretical predictions on the electron and neutron EDMs at one-loop level already exceed the present experimental upper bound. In order to make the theoretical prediction consistent with the experimental data, one can generally adopt three approaches. One possibility is to make the CP phases sufficiently small, i.e.  $\leq 10^{-2}$  [18]. One can also assume a mass suppression by making the supersymmetry spectra heavy, i.e. in the several TeV range [19], or invoke a cancellation among the different contributions to the fermion EDMs [20]. Since the lepton EDM is an interesting topic in both theoretical and experimental aspects, we as well present the lepton EDM by keeping all possible CP violating phases.

This paper is composed by the sections as follows. In Section 2, we introduce the effective Lagrangian method and our notations. Then we will demonstrate how to obtain the supersymmetric two-loop corrections to the lepton MDMs and EDMs. Section 3 is devoted to the numerical analysis and discussion. In Section 4, we give our conclusion. Some tedious formulae are collected in Appendix A.

## 2. Notations and two-loop supersymmetric corrections

The lepton MDMs and EDMs can actually be expressed as the operators

$$\begin{aligned}\mathcal{L}_{\text{MDM}} &= \frac{e}{4m_l} a_l \bar{l} \sigma^{\mu\nu} l F_{\mu\nu}, \\ \mathcal{L}_{\text{EDM}} &= -\frac{i}{2} d_l \bar{l} \sigma^{\mu\nu} \gamma_5 l F_{\mu\nu}.\end{aligned}\quad (8)$$

Here,  $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ ,  $l$  denotes the lepton fermion,  $F_{\mu\nu}$  is the electromagnetic field strength,  $m_l$  is the lepton mass and  $e$  represents the electric charge, respectively. Note that the lepton here is on-shell.

In fact, it is convenient to get the corrections from loop diagrams to lepton MDMs and EDMs in terms of the effective Lagrangian method, if the masses of internal lines are much heavier than the external lepton mass. Assuming external leptons as well as photon are all off-shell, we expand the amplitude of the corresponding triangle diagrams according to the external momenta of leptons and photon. After matching between the effective theory and the full theory, we can get all high dimension operators together with their coefficients. As discussed in Section 1, it is enough to retain only those dimension 6 operators in later calculations:

$$\begin{aligned}\mathcal{O}_1^\mp &= \frac{1}{(4\pi)^2} \bar{l} (i\not{D})^3 \omega_\mp l, \\ \mathcal{O}_2^\mp &= \frac{eQ_f}{(4\pi)^2} \overline{(i\mathcal{D}_\mu l)} \gamma^\mu F \cdot \sigma \omega_\mp l, \\ \mathcal{O}_3^\mp &= \frac{eQ_f}{(4\pi)^2} \bar{l} F \cdot \sigma \gamma^\mu \omega_\mp (i\mathcal{D}_\mu l), \\ \mathcal{O}_4^\mp &= \frac{eQ_f}{(4\pi)^2} \bar{l} (\partial^\mu F_{\mu\nu}) \gamma^\nu \omega_\mp l, \\ \mathcal{O}_5^\mp &= \frac{m_l}{(4\pi)^2} \bar{l} (i\not{D})^2 \omega_\mp l, \\ \mathcal{O}_6^\mp &= \frac{eQ_f m_l}{(4\pi)^2} \bar{l} F \cdot \sigma \omega_\mp l,\end{aligned}\quad (9)$$

with  $\mathcal{D}_\mu = \partial_\mu + ieA_\mu$  and  $\omega_\mp = (1 \mp \gamma_5)/2$ . When the equations of motion are applied to the incoming and outgoing leptons separately, only the operators  $\mathcal{O}_{2,3,6}^\mp$  actually contribute to the MDMs and EDMs of leptons. We will only present the Wilson coefficients of the operators  $\mathcal{O}_{2,3,6}^\mp$  in the effective Lagrangian in our following narration because of the reason mentioned above.

If the full theory is invariant under the combined transformation of charge conjugation, parity and time reversal (CPT), the induced effective theory preserves the symmetry after the heavy freedoms are integrated out. The fact implies the Wilson coefficients of the operators  $\mathcal{O}_{2,3,6}^\mp$

satisfying the relations

$$C_2^\mp = C_3^{\mp*}, \quad C_6^+ = C_6^{-*}, \quad (10)$$

where  $C_i^\mp$  ( $i = 1, 2, \dots, 6$ ) represent the Wilson coefficients of the corresponding operators  $\mathcal{O}_i^\mp$  in the effective Lagrangian. After applying the equations of motion to the external leptons, we find that the concerned terms in the effective Lagrangian are transformed into

$$\begin{aligned} & C_2^\mp \mathcal{O}_2^\mp + C_2^{\mp*} \mathcal{O}_3^\mp + C_6^+ \mathcal{O}_6^+ + C_6^{+*} \mathcal{O}_6^- \\ & \Rightarrow (C_2^+ + C_2^{-*} + C_6^+) \mathcal{O}_6^+ + (C_2^{+*} + C_2^- + C_6^{+*}) \mathcal{O}_6^- \\ & = \frac{e Q_f m_l}{(4\pi)^2} \{ \Re(C_2^+ + C_2^{-*} + C_6^+) \bar{l} \sigma^{\mu\nu} l + i \Im(C_2^+ + C_2^{-*} + C_6^+) \bar{l} \sigma^{\mu\nu} \gamma_5 l \} F_{\mu\nu}. \end{aligned} \quad (11)$$

Here,  $\Re(\dots)$  denotes the operation to take the real part of a complex number, and  $\Im(\dots)$  denotes the operation to take the imaginary part of a complex number. Applying Eqs. (8) and (11), we finally get

$$\begin{aligned} a_l &= \frac{4 Q_f m_l^2}{(4\pi)^2} \Re(C_2^+ + C_2^{-*} + C_6^+), \\ d_l &= -\frac{2e Q_f m_l}{(4\pi)^2} \Im(C_2^+ + C_2^{-*} + C_6^+). \end{aligned} \quad (12)$$

In other words, the MDM of lepton is proportional to real part of the effective coupling  $C_2^+ + C_2^{-*} + C_6^+$ , as well as the EDM of lepton is proportional to imaginary part of the effective coupling  $C_2^+ + C_2^{-*} + C_6^+$ .

Using the effective Lagrangian method, we present the one-loop supersymmetric contribution to muon MDM in [11] which coincides with the previous result in literature. Since the complication of analysis at two-loop order, we will adopt below a terminology where, for example, the “ $\gamma h_k$ ” contribution means the sum of amplitude from those triangle diagrams (indeed three triangles bound together), in which a closed fermion (chargino/neutralino) loop is attached to the virtual Higgs and photon fields with a real photon attached in all possible ways to the internal lines. Because the sum of amplitude from those “triangle” diagrams corresponding to each “self-energy” obviously respects the Ward identity requested by QED gauge symmetry, we can calculate the contributions of all the “self-energies” separately. Taking the same steps which we did in our earlier works [9–11], we obtain the effective Lagrangian that originates from the self energy diagrams in Fig. 1. In the bare effective Lagrangian from the ‘WW’ and ‘ZZ’ contributions, the ultraviolet divergence caused by divergent subdiagrams can be subtracted safely in on-mass-shell scheme [12]. Now, we present the effective Lagrangian corresponding to the diagrams in Fig. 1 respectively.

### 2.1. The effective Lagrangian from $\gamma h_k$ ( $k = 1, 2, 3$ ) and $\gamma G_0$ sector

As a closed chargino loop is attached to the virtual neutral Higgs and photon fields, a real photon can be emitted from either the virtual lepton or the virtual charginos in the self energy diagram. When a real photon is emitted from the virtual charginos, the corresponding “triangle” diagrams belong to the typical two-loop Bar-Zee-type diagrams [21]. Within the framework of CP violating MSSM, the contributions from two-loop Bar-Zee-type diagrams to the EDMs of

those light fermions are discussed extensively in literature [22]. When a real photon is attached to the internal standard fermion, the correction from corresponding triangle diagram to the effective Lagrangian is zero because of the Furry theorem, this point is also verified through a strict analysis. The corresponding effective Lagrangian from this sector is written as

$$\begin{aligned}\mathcal{L}_{\gamma h_k} = & \frac{e^4 (Z_H)_{1k}}{2\sqrt{2}(4\pi)^2 s_w^2 \Lambda^2 \cos \beta} \left\{ \Re(\mathcal{H}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_1(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm})(\mathcal{O}_6^+ + \mathcal{O}_6^-) \right. \\ & \left. + i \Im(\mathcal{H}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_2(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm})(\mathcal{O}_6^+ - \mathcal{O}_6^-) \right\} \\ & - \frac{e^4 (Z_H)_{3k} \tan \beta}{2\sqrt{2}(4\pi)^2 s_w^2 \Lambda^2} \left\{ \Re(\mathcal{A}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_2(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm}) \right] (\mathcal{O}_6^+ + \mathcal{O}_6^-) \\ & \left. - i \Im(\mathcal{A}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_1(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm})(\mathcal{O}_6^+ - \mathcal{O}_6^-) \right\}\end{aligned}\quad (13)$$

with

$$\begin{aligned}\mathcal{H}_{ij}^k &= (U_R^\dagger)_{i1} (U_L)_{2j} (Z_H)_{1k} + (U_R^\dagger)_{i2} (U_L)_{1j} (Z_H)_{2k}, \\ \mathcal{A}_{ij}^k &= ((U_R^\dagger)_{i1} (U_L)_{2j} \sin \beta + (U_R^\dagger)_{i2} (U_L)_{1j} \cos \beta) (Z_H)_{3k} \quad (i, j = 1, 2).\end{aligned}\quad (14)$$

Where the two unitary matrices  $U_{L,R}$  denote the left- and right-mixing matrices of charginos,  $\Lambda$  denotes the energy scale of new physics, and  $x_i = m_i^2/\Lambda^2$  respectively. We adopt the abbreviations:  $c_w = \cos \theta_w$ ,  $s_w = \sin \theta_w$ , where  $\theta_w$  is the Weinberg angle. The concrete expressions of  $T_{1,2}$  can be found in [Appendix A](#).

Accordingly, the lepton MDMs and EDMs from  $\gamma h_k$  sector are written as

$$\begin{aligned}a_l^{\gamma h_k} &= \frac{\sqrt{2}e^4 Q_f m_l^2 (Z_H)_{1k}}{(4\pi)^4 s_w^2 \Lambda^2 \cos \beta} \Re(\mathcal{H}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_1(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm}) \\ &\quad - \frac{\sqrt{2}e^4 Q_f m_l^2 (Z_H)_{3k} \tan \beta}{(4\pi)^4 s_w^2 \Lambda^2} \Re(\mathcal{A}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_2(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm}), \\ d_l^{\gamma h_k} &= -\frac{e^5 Q_f m_l (Z_H)_{1k}}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2 \cos \beta} \Im(\mathcal{H}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_2(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm}) \\ &\quad - \frac{e^5 Q_f m_l (Z_H)_{3k} \tan \beta}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2} \Im(\mathcal{A}_{ii}^k) \left( \frac{x_{\chi_i^\pm}}{x_w} \right)^{1/2} T_1(x_{h_k}, x_{\chi_i^\pm}, x_{\chi_i^\pm}),\end{aligned}\quad (15)$$

which are enhanced by large  $\tan \beta$ . Note here that the corrections from this sector to the MDM of lepton depend on a linear combination of real parts of the effective couplings  $\mathcal{H}_{ii}^k$  and  $\mathcal{A}_{ii}^k$ , and the corrections from this sector to the EDM of lepton depend on a linear combination of imaginary parts of the effective couplings  $\mathcal{H}_{ii}^k$  and  $\mathcal{A}_{ii}^k$ . In the limit  $x_{\chi_i^\pm} \gg x_{h_k}$ , the above expressions can be simplified as

$$\begin{aligned}
a_l^{\gamma h_k} &= -\frac{\sqrt{2}e^4 Q_f m_l^2 (Z_H)_{1k}}{(4\pi)^4 s_w^2 \Lambda^2 \cos \beta} \Re(\mathcal{H}_{ii}^k) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \\
&\quad - \frac{\sqrt{2}e^4 Q_f m_l^2 (Z_H)_{3k} \tan \beta}{(4\pi)^4 s_w^2 \Lambda^2} \Re(\mathcal{A}_{ii}^k) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \\
&\quad \times \left[ \frac{\ln x_{h_k}}{x_{\chi_i^\pm}} + \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right], \\
d_l^{\gamma h_k} &= -\frac{e^5 Q_f m_l (Z_H)_{1k}}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2 \cos \beta} \Im(\mathcal{H}_{ii}^k) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \left[ \frac{\ln x_{h_k}}{x_{\chi_i^\pm}} + \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \\
&\quad + \frac{e^5 Q_f m_l (Z_H)_{3k} \tan \beta}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2} \Im(\mathcal{A}_{ii}^k) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}). \quad (16)
\end{aligned}$$

Similarly, we can formulate the corrections from  $\gamma G_0$  sector to the effective Lagrangian as

$$\begin{aligned}
\mathcal{L}_{\gamma G} &= \frac{e^4}{2\sqrt{2}(4\pi)^2 s_w^2 \Lambda^2} \left\{ \Re(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} T_2(x_Z, x_{\chi_i^\pm}, x_{\chi_i^\pm}) (\mathcal{O}_6^+ + \mathcal{O}_6^-) \right. \\
&\quad \left. - i \Im(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} T_1(x_Z, x_{\chi_i^\pm}, x_{\chi_i^\pm}) (\mathcal{O}_6^+ - \mathcal{O}_6^-) \right\}, \quad (17)
\end{aligned}$$

with

$$\mathcal{B}_{ij} = -(U_R^\dagger)_{i1} (U_L)_{2j} \cos \beta + (U_R^\dagger)_{i2} (U_L)_{1j} \sin \beta \quad (i, j = 1, 2). \quad (18)$$

Correspondingly, the corrections to the lepton MDMs and EDMs from this sector are:

$$\begin{aligned}
a_l^{\gamma G} &= \frac{\sqrt{2}e^4 Q_f m_l^2}{(4\pi)^4 s_w^2 \Lambda^2} \Re(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} T_2(x_Z, x_{\chi_i^\pm}, x_{\chi_i^\pm}), \\
d_l^{\gamma G} &= \frac{e^5 Q_f m_l}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2} \Im(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} T_1(x_Z, x_{\chi_i^\pm}, x_{\chi_i^\pm}). \quad (19)
\end{aligned}$$

The corrections from this sector to the MDM of lepton are proportional to real parts of the effective couplings  $\mathcal{B}_{ii}$ , and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings  $\mathcal{B}_{ii}$ , separately. In the limit  $x_{\chi_i^\pm} \gg x_Z$ , we have

$$\begin{aligned}
a_l^{\gamma G} &= \frac{\sqrt{2}e^4 Q_f m_l^2}{(4\pi)^4 s_w^2 \Lambda^2} \Re(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \left[ \frac{\ln x_Z}{x_{\chi_i^\pm}} + \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right], \\
d_l^{\gamma G} &= \frac{e^5 Q_f m_l}{\sqrt{2}(4\pi)^4 s_w^2 \Lambda^2} \Im(\mathcal{B}_{ii}) \left(\frac{x_{\chi_i^\pm}}{x_w}\right)^{1/2} \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \frac{\partial}{\partial x_{\chi_j^\pm}} \varphi_1(x_{\chi_i^\pm}, x_{\chi_j^\pm}). \quad (20)
\end{aligned}$$

Using the concrete expression of  $\varphi_1(x, y)$  presented in [Appendix A](#), one can verify easily that the corrections to the lepton MDMs and EDMs from the sectors are suppressed by the masses of charginos as  $m_{\chi_i^\pm} \gg m_{h_k}, m_Z$  ( $i = 1, 2$ ).



## 2.2. The effective Lagrangian from $Zh_k$ ( $ZG_0$ ) sector

As a closed chargino loop is attached to the virtual Higgs and  $Z$  gauge boson fields, a real photon can be attached to either the virtual lepton or the virtual charginos in the self energy diagram. When a real photon is attached to the virtual lepton, the corresponding amplitude only modifies the Wilson coefficients of the operators  $\mathcal{O}_5^\pm$  in the effective Lagrangian after the heavy freedoms are integrated out. In other words, this triangle diagram does not contribute to the lepton MDMs and EDMs. A real photon can be only attached to the virtual lepton as the closed loop is composed of neutralinos, the corresponding triangle diagram does not affect the theoretical predictions on the lepton MDMs and EDMs for the same reason. Considering the points above, we formulate the contributions from  $Zh_0$  sector to the effective Lagrangian as

$$\begin{aligned}
\mathcal{L}_{Zh_k} = & -\frac{e^4(Z_H)_{1k}}{16\sqrt{2}(4\pi)^2 s_w^4 c_w^2 Q_f \Lambda^2 \cos \beta} (T_f^Z - 2Q_f s_w^2) \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [(4 + 2 \ln x_{\chi_j^\pm}) \right. \\
& \times \mathcal{Q}_{0,1}(x_z, x_{h_k}) + F_1(x_z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm})] \Re(\mathcal{H}_{ji}^k \xi_{ij}^L + \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) (\mathcal{O}_6^+ + \mathcal{O}_6^-) \\
& + i \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [-2(\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) \mathcal{Q}_{0,1}(x_z, x_{h_k}) + F_1(x_z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \\
& + F_2(x_z, x_{h_k}, x_{\chi_j^\pm}, x_{\chi_i^\pm})] \Im(\mathcal{H}_{ji}^k \xi_{ij}^L - \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) (\mathcal{O}_6^- - \mathcal{O}_6^+) \Big\} \\
& + \frac{e^4(Z_H)_{3k} \tan \beta}{16\sqrt{2}(4\pi)^2 s_w^4 c_w^2 Q_f \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left\{ -i \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [2(2 + \ln x_{\chi_j^\pm}) \right. \\
& \times \mathcal{Q}_{0,1}(x_z, x_{h_k}) + F_1(x_z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm})] \Im(\mathcal{A}_{ji}^k \xi_{ij}^L + \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R) (\mathcal{O}_6^- - \mathcal{O}_6^+) \\
& + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [-2(\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) \mathcal{Q}_{0,1}(x_z, x_{h_k}) + F_1(x_z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \\
& + F_2(x_z, x_{h_k}, x_{\chi_j^\pm}, x_{\chi_i^\pm})] \Re(\mathcal{A}_{ji}^k \xi_{ij}^L - \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R) (\mathcal{O}_6^- + \mathcal{O}_6^+) \Big\} + \dots \quad (21)
\end{aligned}$$

with

$$\begin{aligned}
\xi_{ij}^L &= 2\delta_{ij} \cos 2\theta_w + (U_L^\dagger)_{i1} (U_L)_{1j}, \\
\xi_{ij}^R &= 2\delta_{ij} \cos 2\theta_w + (U_R^\dagger)_{i1} (U_R)_{1j} \quad (i, j = 1, 2), \quad (22)
\end{aligned}$$

where the concrete expressions of the functions  $\mathcal{Q}_{i,j}(x_1, x_2)$ ,  $F_{1,2}(x_1, x_2, x_3, x_4)$  are listed in [Appendix A](#). Additional,  $T_f^Z$  is the isospin of lepton, and  $Q_f$  is the electric charge of lepton, respectively. Using Eq. (21), we get the corrections to the lepton MDMs and EDMs from  $Zh_k$  sector as

$$\begin{aligned}
a_l^{Zh_k} = & -\frac{e^4 m_l^2 (Z_H)_{1k}}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2 \cos \beta} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [2(2 + \ln x_{\chi_j^\pm}) \mathcal{Q}_{i,j}(x_z, x_{h_k}) \\
& + F_1(x_z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm})] \Re(\mathcal{H}_{ji}^k \xi_{ij}^L + \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) \\
& + \frac{e^4 m_l^2 (Z_H)_{3k} \tan \beta}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} [-2(\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) \mathcal{Q}_{0,1}(x_z, x_{h_k})
\end{aligned}$$

$$\begin{aligned}
& + F_1(x_Z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm}) + F_2(x_Z, x_{h_k}, x_{\chi_j^\pm}, x_{\chi_i^\pm}) \Big] \Re(\mathcal{A}_{ji}^k \xi_{ij}^L - \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R), \\
d_l^{Zh_k} = & \frac{e^5 m_l (Z_H)_{1k}}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2 \cos \beta} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ 2(\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) \right. \\
& \times \mathcal{Q}_{0,1}(x_Z, x_{h_k}) - F_1(x_Z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm}) - F_2(x_Z, x_{h_k}, x_{\chi_j^\pm}, x_{\chi_i^\pm}) \Big] \\
& \times \Im(\mathcal{H}_{ji}^k \xi_{ij}^L - \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) - \frac{e^5 m_l (Z_H)_{3k} \tan \beta}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \\
& \times \left[ 2(2 + \ln x_{\chi_j^\pm}) \mathcal{Q}_{i,j}(x_Z, x_{h_k}) + F_1(x_Z, x_{h_k}, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \Im(\mathcal{A}_{ji}^k \xi_{ij}^L + \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R). \quad (23)
\end{aligned}$$

The above equations contain the suppression factor  $1 - 4s_w^2$  because  $Q_f = -1$  and  $T_f^Z = -1/2$  for charged leptons. The corrections from this sector to the MDM of lepton are decided by a linear combination of real parts of the effective couplings  $\mathcal{H}_{ji}^k \xi_{ij}^L + \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R$  and  $\mathcal{A}_{ji}^k \xi_{ij}^L - \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R$ , and the corrections from this sector to the EDM of lepton are decided by a linear combination of imaginary parts of the effective couplings  $\mathcal{H}_{ji}^k \xi_{ij}^L - \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R$  and  $\mathcal{A}_{ji}^k \xi_{ij}^L + \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R$ . In the limit  $x_{\chi_i^\pm}, x_{\chi_j^\pm} \gg x_Z, x_{h_k}$ , Eq. (23) can be approximated as

$$\begin{aligned}
a_l^{Zh_k} = & -\frac{e^4 m_l^2 (Z_H)_{1k}}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2 \cos \beta} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. - \frac{2 - 2x_{\chi_i^\pm} \mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm})}{x_{\chi_i^\pm} - x_{\chi_j^\pm}} \cdot \mathcal{Q}_{1,1}(x_Z, x_{h_k}) \right] \Re(\mathcal{H}_{ji}^k \xi_{ij}^L + \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) \\
& + \frac{e^4 m_l^2 (Z_H)_{3k} \tan \beta}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \left( \frac{\partial \varphi_1}{\partial x_{\chi_i^\pm}} + \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}} \right) (x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. + 2\mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \mathcal{Q}_{1,1}(x_Z, x_{h_k}) \right] \Re(\mathcal{A}_{ji}^k \xi_{ij}^L - \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R), \\
d_l^{Zh_k} = & -\frac{e^5 m_l (Z_H)_{1k}}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2 \cos \beta} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \left( \frac{\partial \varphi_1}{\partial x_{\chi_i^\pm}} + \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}} \right) \right. \\
& \times (x_{\chi_i^\pm}, x_{\chi_j^\pm}) + 2\mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \mathcal{Q}_{1,1}(x_Z, x_{h_k}) \Big] \Im(\mathcal{H}_{ji}^k \xi_{ij}^L - \mathcal{H}_{ji}^{k,\dagger} \xi_{ij}^R) \\
& - \frac{e^5 m_l (Z_H)_{3k} \tan \beta}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. - \frac{2 - 2x_{\chi_i^\pm} \mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm})}{x_{\chi_i^\pm} - x_{\chi_j^\pm}} \cdot \mathcal{Q}_{1,1}(x_Z, x_{h_k}) \right] \Im(\mathcal{A}_{ji}^k \xi_{ij}^L + \mathcal{A}_{ji}^{k,\dagger} \xi_{ij}^R). \quad (24)
\end{aligned}$$

Similarly, the contribution from  $ZG_0$  sector to the effective Lagrangian is

$$\begin{aligned}
\mathcal{L}_{ZG_0} = & -\frac{e^4}{16\sqrt{2}(4\pi)^2 s_w^4 c_w^2 Q_f \Lambda^2} \left\{ -i \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \frac{2}{x_Z} (2 + \ln x_{\chi_j^\pm}) \right. \right. \\
& \left. \left. + F_1(x_Z, x_Z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \Im(\mathcal{B}_{ji} \xi_{ij}^L + \mathcal{B}_{ji}^\dagger \xi_{ij}^R) (T_f^Z - 2Q_f s_w^2) (\mathcal{O}_6^- - \mathcal{O}_6^+) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ -\frac{2}{x_z} (\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) + F_1(x_z, x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. + F_2(x_z, x_z, x_{\chi_j^\pm}, x_{\chi_i^\pm}) \right] \Re(\mathcal{B}_{ji} \xi_{ij}^L - \mathcal{B}_{ji}^\dagger \xi_{ij}^R) (T_f^Z - 2Q_f s_w^2) (\mathcal{O}_6^- + \mathcal{O}_6^+) \Big\} + \dots,
\end{aligned} \quad (25)$$

and the contributions to the lepton MDMs and EDMs are:

$$\begin{aligned}
a_l^{ZG} &= -\frac{e^4 m_l^2}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ -\frac{2}{x_z} (\ln x_{\chi_i^\pm} - \ln x_{\chi_j^\pm}) \right. \\
& \left. + F_1(x_z, x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) + F_2(x_z, x_z, x_{\chi_j^\pm}, x_{\chi_i^\pm}) \right] \Re(\mathcal{B}_{ji} \xi_{ij}^L - \mathcal{B}_{ji}^\dagger \xi_{ij}^R), \\
d_l^{ZG} &= \frac{e^5 m_l}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \frac{2}{x_z} (2 + \ln x_{\chi_j^\pm}) \right. \\
& \left. + F_1(x_z, x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \Im(\mathcal{B}_{ji} \xi_{ij}^L + \mathcal{B}_{ji}^\dagger \xi_{ij}^R).
\end{aligned} \quad (26)$$

Here, the corrections from this sector to the MDM of lepton are proportional to real parts of the effective couplings  $\mathcal{B}_{ji} \xi_{ij}^L - \mathcal{B}_{ji}^\dagger \xi_{ij}^R$ , and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings  $\mathcal{B}_{ji} \xi_{ij}^L + \mathcal{B}_{ji}^\dagger \xi_{ij}^R$ . When  $x_{\chi_i^\pm}, x_{\chi_j^\pm} \gg x_z$ , Eq. (26) can be approached by

$$\begin{aligned}
a_l^{ZG} &= -\frac{e^4 m_l^2}{4\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \left( \frac{\partial \varphi_1}{\partial x_{\chi_i^\pm}} + \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}} \right) (x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. + 2(1 + \ln x_z) \mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \Re(\mathcal{B}_{ji} \xi_{ij}^L - \mathcal{B}_{ji}^\dagger \xi_{ij}^R), \\
d_l^{ZG} &= \frac{e^5 m_l}{8\sqrt{2}(4\pi)^4 s_w^4 c_w^2 \Lambda^2} (T_f^Z - 2Q_f s_w^2) \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} \left[ \frac{\partial \varphi_1}{\partial x_{\chi_j^\pm}} (x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
& \left. - \frac{2 - 2x_{\chi_i^\pm} \mathcal{Q}_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm})}{x_{\chi_i^\pm} - x_{\chi_j^\pm}} \cdot (1 + \ln x_z) \right] \Im(\mathcal{B}_{ji} \xi_{ij}^L + \mathcal{B}_{ji}^\dagger \xi_{ij}^R).
\end{aligned} \quad (27)$$

### 2.3. The effective Lagrangian from $\gamma Z$ sector

When a closed chargino loop is attached to the virtual  $\gamma$  and  $Z$  gauge bosons, the corresponding correction to the effective Lagrangian is very tedious. If we ignore the terms which are proportional to the suppression factor  $1 - 4s_w^2$ , the correction from this sector to the effective Lagrangian is drastically simplified as

$$\begin{aligned}
\mathcal{L}_{\gamma Z} &= \frac{e^4}{8(4\pi)^2 s_w^2 c_w^2 \Lambda^2} (\xi_{ii}^L - \xi_{ii}^R) \lim_{x_{\chi_i^\pm} \rightarrow x_{\chi_j^\pm}} T_3(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \\
& \times [(T_f^Z - Q_f s_w^2) (\mathcal{O}_2^- + \mathcal{O}_3^-) + Q_f s_w^2 (\mathcal{O}_2^+ + \mathcal{O}_3^+)] + \dots
\end{aligned} \quad (28)$$

Using the definitions of the matrices  $\xi_{ij}^{L,R}$  in Eq. (22), one can find that the effective couplings  $\xi_{ii}^L - \xi_{ii}^R$  ( $i = 1, 2$ ) are real. Correspondingly, the correction to the lepton MDMs from this sector

is written as

$$a_l^{\gamma Z} = \frac{e^4 Q_f m_l^2}{4(4\pi)^4 s_w^2 c_w^2 \Lambda^2} (\xi_{ii}^L - \xi_{ii}^R) \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} T_3(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}), \quad (29)$$

and the correction to the lepton EDMs is zero. In the limit  $x_{\chi_i^\pm} \gg x_z$ , we can approximate the correction to the lepton MDMs from this sector as

$$a_l^{\gamma Z} = \frac{e^4 Q_f m_l^2}{4(4\pi)^4 s_w^2 c_w^2 \Lambda^2} (\xi_{ii}^L - \xi_{ii}^R) \left[ \frac{13}{18x_{\chi_i^\pm}} + \frac{\ln x_{\chi_i^\pm} - 2 \ln x_z}{3x_{\chi_i^\pm}} \right. \\ \left. + \lim_{x_{\chi_j^\pm} \rightarrow x_{\chi_i^\pm}} \left( 2x_{\chi_i^\pm} \frac{\partial^2 \varphi_1}{\partial x_{\chi_i^\pm}^2} - \frac{\partial \varphi_1}{\partial x_{\chi_i^\pm}} \right) (x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right]. \quad (30)$$

#### 2.4. The effective Lagrangian from $W^\mp H^\pm$ ( $W^\mp G^\pm$ ) sector

As a closed chargino–neutralino loop is attached to the virtual  $W^\pm$  gauge boson and charged Higgs  $H^\mp$ , the induced Lagrangian can be written as

$$\mathcal{L}_{WH} = -\frac{e^4 \tan \beta}{16(4\pi)^2 s_w^4 c_w Q_f \Lambda^2} \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_3(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) [(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L \right. \\ - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \mathcal{O}_6^- + (\sin \beta (\mathcal{G}^{1L})_{ij}^\dagger (\zeta^L)_{ji}^\dagger - \cos \beta (\mathcal{G}^{1R})_{ij}^\dagger (\zeta^R)_{ji}^\dagger) \mathcal{O}_6^+] \\ + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_4(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) [(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \mathcal{O}_6^- \\ + (\sin \beta (\mathcal{G}^{1L})_{ij}^\dagger (\zeta^R)_{ji}^\dagger - \cos \beta (\mathcal{G}^{1R})_{ij}^\dagger (\zeta^L)_{ji}^\dagger) \mathcal{O}_6^+] \\ + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_5(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) [(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \mathcal{O}_6^- \\ + (\sin \beta (\mathcal{G}^{1L})_{ij}^\dagger (\zeta^L)_{ji}^\dagger + \cos \beta (\mathcal{G}^{1R})_{ij}^\dagger (\zeta^R)_{ji}^\dagger) \mathcal{O}_6^+] \\ + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_6(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) [(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \mathcal{O}_6^- \\ \left. + (\sin \beta (\mathcal{G}^{1L})_{ij}^\dagger (\zeta^R)_{ji}^\dagger + \cos \beta (\mathcal{G}^{1R})_{ij}^\dagger (\zeta^L)_{ji}^\dagger) \mathcal{O}_6^+] \right\} \quad (31)$$

with

$$\zeta_{ij}^L = \mathcal{N}_{i2}^\dagger (U_L)_{1j} - \frac{1}{\sqrt{2}} \mathcal{N}_{i4}^\dagger (U_L)_{2j}, \\ \zeta_{ij}^R = \mathcal{N}_{2i} (U_R^\dagger)_{j1} + \frac{1}{\sqrt{2}} \mathcal{N}_{3i} (U_R^\dagger)_{j2}, \\ \mathcal{G}_{ji}^{1L} = \frac{1}{\sqrt{2}} (U_L)_{2j} (\mathcal{N}_{1i} s_w + \mathcal{N}_{2i} c_w) - (U_L)_{1j} \mathcal{N}_{3i} c_w, \\ \mathcal{G}_{ji}^{1R} = \frac{1}{\sqrt{2}} (U_R^\dagger)_{j2} (\mathcal{N}_{i1}^\dagger s_w + \mathcal{N}_{i2}^\dagger c_w) - (U_R^\dagger)_{j1} \mathcal{N}_{i4}^\dagger c_w, \\ (i = 1, \dots, 4, j = 1, 2). \quad (32)$$

Here, the  $4 \times 4$  matrix  $\mathcal{N}$  denotes the mixing matrix of the four neutralinos  $\chi_i^0$  ( $i = 1, \dots, 4$ ).

The corresponding corrections to the lepton MDMs and EDMs are respectively expressed as

$$\begin{aligned}
 a_l^{WH} = & -\frac{e^4 m_l^2 \tan \beta}{4(4\pi)^4 s_w^4 c_w \Lambda^2} \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_3(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \right. \\
 & \times \Re(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
 & + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_4(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \\
 & + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_5(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
 & \left. + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_6(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \right\}, \\
 d_l^{WH} = & -\frac{e^5 m_l \tan \beta}{8(4\pi)^4 s_w^4 c_w \Lambda^2} \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_3(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \right. \\
 & \times \Im(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
 & + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_4(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \\
 & + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_5(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
 & \left. + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_6(x_w, x_{H^\pm}, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \right\}, \quad (33)
 \end{aligned}$$

where the concrete expressions of  $F_{3,4,5,6}$  can be found in [Appendix A](#). The corrections from this sector to the MDM of lepton are decided by a linear combination of real parts of the effective couplings  $\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R$ ,  $\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L$ ,  $\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R$ , as well as  $\sin \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \cos \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L$ , and the corrections from this sector to the EDM of lepton are decided by a linear combination of imaginary parts of those effective couplings. Using the asymptotic expansion of the two-loop vacuum integral  $\Phi(x, y, z)$  presented in [Appendix A](#), we can simplify the expressions of Eq. (33) in the limit  $x_{\chi_i^0}, x_{\chi_j^\pm} \gg x_w$ .

As a closed chargino–neutralino loop is attached to the virtual  $W^\pm$  gauge boson and charged Goldstone  $G^\mp$ , the corresponding corrections to the lepton MDMs and EDMs are similarly formulated as

$$\begin{aligned}
 a_l^{WG} = & -\frac{e^4 m_l^2}{4(4\pi)^4 s_w^4 c_w \Lambda^2} \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_3(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \right. \\
 & + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_4(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \\
 & + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_5(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
 & \left. + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_6(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Re(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \right\},
 \end{aligned}$$

$$\begin{aligned}
d_l^{WG} = & -\frac{e^5 m_l}{8(4\pi)^4 s_w^4 c_w \Lambda^2} \left\{ \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_3(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \right. \\
& + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_4(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \\
& + \left( \frac{x_{\chi_j^\pm}}{x_w} \right)^{1/2} F_5(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R) \\
& \left. + \left( \frac{x_{\chi_i^0}}{x_w} \right)^{1/2} F_6(x_w, x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \Im(\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L) \right\}. \quad (34)
\end{aligned}$$

Similarly, the corrections from this sector to the MDM of lepton depend on a linear combination of real parts of the effective couplings  $\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R$ ,  $\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R + \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L$ ,  $\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^L - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^R$ , as well as  $\cos \beta \mathcal{G}_{ji}^{1L} \zeta_{ij}^R - \sin \beta \mathcal{G}_{ji}^{1R} \zeta_{ij}^L$ , and the corrections from this sector to the EDM of lepton depend on a linear combination of imaginary parts of those effective couplings.

The contributions from those above sectors to effective Lagrangian do not contain ultraviolet divergence. In the pieces discussed below, the coefficients of high dimensional operators in effective Lagrangian contain ultraviolet divergence that is caused by the divergent subdiagrams. In order to obtain physical predictions of lepton MDMs and EDMs, it is necessary to adopt a concrete renormalization scheme removing the ultraviolet divergence. In literature, the on-shell renormalization scheme is adopted frequently to subtract the ultraviolet divergence which appears in the radiative electroweak corrections [12]. As an over-subtract scheme, the counter terms include some finite terms which originate from those renormalization conditions in the on-shell scheme beside the ultraviolet divergence to cancel the corresponding ultraviolet divergence contained by the bare Lagrangian. In the concrete calculation performed here, we apply this scheme to subtract the ultraviolet divergence caused by the divergent subdiagrams.

### 2.5. The effective Lagrangian from the ZZ sector

The self energy of Z gauge boson composed of a closed chargino loop induces the ultraviolet divergence in the Wilson coefficients of effective Lagrangian. Generally, the unrenormalized self energy of the weak gauge boson Z can be written as

$$\Sigma_{\mu\nu}^Z(p) = \Lambda^2 A_0^z g_{\mu\nu} + \left( A_1^z + \frac{p^2}{\Lambda^2} A_2^z \right) (p^2 g_{\mu\nu} - p_\mu p_\nu) + \left( B_1^z + \frac{p^2}{\Lambda^2} B_2^z \right) p_\mu p_\nu. \quad (35)$$

Correspondingly, the counter terms are given as

$$\Sigma_{\mu\nu}^{ZC}(p) = -(\delta m_Z^2 + m_Z^2 \delta Z_Z) g_{\mu\nu} - \delta Z_Z (p^2 g_{\mu\nu} - p_\mu p_\nu). \quad (36)$$

The renormalized self energy is given by

$$\hat{\Sigma}_{\mu\nu}^Z(p) = \Sigma_{\mu\nu}^Z(p) + \Sigma_{\mu\nu}^{ZC}(p). \quad (37)$$

For on-shell external gauge boson Z, we have [12]

$$\begin{aligned}
\hat{\Sigma}_{\mu\nu}^Z(p) \epsilon^\nu(p) \Big|_{p^2=m_Z^2} &= 0, \\
\lim_{p^2 \rightarrow m_Z^2} \frac{1}{p^2 - m_Z^2} \hat{\Sigma}_{\mu\nu}^Z(p) \epsilon^\nu(p) &= \epsilon_\mu(p), \quad (38)
\end{aligned}$$

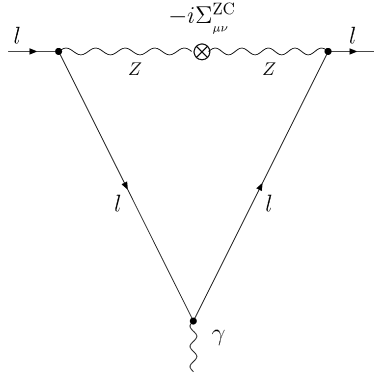


Fig. 2. The counter term diagram to cancel the ultraviolet caused by the self energy of Z boson.

where  $\epsilon(p)$  is the polarization vector of Z gauge boson. From Eq. (38), we get the counter terms

$$\begin{aligned}\delta Z_z &= A_1^z + \frac{m_z^2}{\Lambda^2} A_2^z = A_1^z + x_z A_2^z, \\ \delta m_z^2 &= A_0^z \Lambda^2 - m_z^2 \delta Z_z.\end{aligned}\quad (39)$$

Accordingly, the effective Lagrangian originating from the counter term diagram (Fig. 2) can be formulated as

$$\begin{aligned}\delta \mathcal{L}_{ZZ}^C &= -\frac{e^4}{12(4\pi)^2 s_w^4 c_w^4 \Lambda^2} (4\pi x_R)^{2\epsilon} \frac{\Gamma^2(1+\epsilon)}{(1-\epsilon)^2} \left\{ (\xi_{ji}^L \xi_{ij}^L + \xi_{ji}^R \xi_{ij}^R) \left[ -\frac{1}{\epsilon} \frac{x_{\chi_i}^\pm + x_{\chi_j}^\pm}{x_z^2} \right. \right. \\ &\quad \left. \left. + \frac{5(x_{\chi_i}^\pm + x_{\chi_j}^\pm)}{12x_z^2} + \frac{\mathcal{Q}_{2,1}(x_{\chi_i}^\pm, x_{\chi_j}^\pm)}{x_z^2} + \frac{5}{12x_z} + \frac{x_{\chi_i}^\pm + x_{\chi_j}^\pm}{x_z^2} \ln x_R \right] \right. \\ &\quad \left. + 2(x_{\chi_i}^\pm x_{\chi_j}^\pm)^{1/2} (\xi_{ji}^L \xi_{ij}^R + \xi_{ji}^R \xi_{ij}^L) \left[ \frac{1}{\epsilon x_z^2} - \frac{\mathcal{Q}_{1,1}(x_{\chi_i}^\pm, x_{\chi_j}^\pm)}{x_z^2} + \frac{1}{12x_z^2} - \frac{\ln x_R}{x_z^2} \right] \right\} \\ &\quad \times [(T_f^Z - Q_f s_w^2)(\mathcal{O}_2^- + \mathcal{O}_3^-) + Q_f^2 s_w^4 (\mathcal{O}_2^+ + \mathcal{O}_3^+)] \\ &\quad + \frac{e^4}{4(4\pi)^2 s_w^4 c_w^4 \Lambda^2} (4\pi x_R)^{2\epsilon} \frac{\Gamma^2(1+\epsilon)}{(1-\epsilon)^2} \left\{ (\xi_{ji}^L \xi_{ij}^L + \xi_{ji}^R \xi_{ij}^R) \left[ \frac{1}{\epsilon} \frac{x_{\chi_i}^\pm + x_{\chi_j}^\pm}{x_z^2} \right. \right. \\ &\quad \left. \left. - \frac{\mathcal{Q}_{2,1}(x_{\chi_i}^\pm, x_{\chi_j}^\pm)}{x_z^2} - \frac{x_{\chi_i}^\pm + x_{\chi_j}^\pm}{x_z^2} \left( \frac{7}{2} + \ln x_l - \ln x_z \right) + \frac{1}{4x_z} - \frac{x_{\chi_i}^\pm + x_{\chi_j}^\pm}{x_z^2} \ln x_R \right] \right. \\ &\quad \left. + 2(x_{\chi_i}^\pm x_{\chi_j}^\pm)^{1/2} (\xi_{ji}^L \xi_{ij}^R + \xi_{ji}^R \xi_{ij}^L) \left[ -\frac{1}{\epsilon x_z^2} + \frac{\mathcal{Q}_{1,1}(x_{\chi_i}^\pm, x_{\chi_j}^\pm)}{x_z^2} \right. \right. \\ &\quad \left. \left. + \frac{1}{x_z^2} (3 + \ln x_l - \ln x_z) + \frac{\ln x_R}{x_z^2} \right] \right\} \mathcal{Q}_f s_w^2 (T_f^Z - Q_l s_w^2) (\mathcal{O}_6^- + \mathcal{O}_6^+) + \dots \quad (40)\end{aligned}$$

Here,  $\epsilon = 2 - D/2$  with  $D$  representing the time-space dimension, and  $x_R = \Lambda_{RE}^2/\Lambda^2$  ( $\Lambda_{RE}$  denotes the renormalization scale).

As a result of the preparation mentioned above, we can add the contributions from the counter term diagram to cancel the corresponding ultraviolet divergence contained by the bare effective

Lagrangian. Using the definitions of the matrices  $\xi_{ij}^{L,R}$  in Eq. (22), we derive  $\xi_{ij}^L = \xi_{ji}^{L*}$ ,  $\xi_{ij}^R = \xi_{ji}^{R*}$ . The resulted theoretical predictions on the lepton MDMs and EDMs are respectively written as

$$\begin{aligned}
 a_{l,\chi^\pm}^{ZZ} = & -\frac{e^4 m_l^2}{(4\pi)^4 s_w^4 c_w^4 \Lambda^2} \left\{ (|\xi_{ij}^L|^2 + |\xi_{ij}^R|^2) [(T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4] \right. \\
 & \times \left[ \frac{Q_f}{3} \left( T_5(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) + \frac{x_{\chi_i^\pm} + x_{\chi_j^\pm}}{x_z^2} \ln x_R \right) + \frac{1}{4} T_4(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right] \\
 & + \frac{1}{8} (|\xi_{ij}^L|^2 - |\xi_{ij}^R|^2) [(T_f^Z - Q_f s_w^2)^2 - Q_f^2 s_w^4] T_6(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \\
 & - \Re(\xi_{ij}^L \xi_{ji}^R) [(T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4] (x_{\chi_i^\pm} x_{\chi_j^\pm})^{1/2} \\
 & \times \left[ \frac{1}{4} T_7(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) + \frac{4Q_f}{3x_z^2} \ln \frac{x_z}{x_R} - \frac{7Q_f}{3x_z^2} \right] \\
 & - (|\xi_{ij}^L|^2 + |\xi_{ij}^R|^2) s_w^2 (T_f^Z - Q_f s_w^2) \left[ \frac{Q_f}{4} T_9(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right. \\
 & - \frac{Q_f^2}{4x_z} + \frac{Q_f^2}{x_z^2} \left( 2 - \ln \frac{x_z}{x_R} \right) (x_{\chi_i^\pm} + x_{\chi_j^\pm}) - \frac{Q_f^2}{2x_z^2} (x_{\chi_i^\pm} \ln x_{\chi_i^\pm} + x_{\chi_j^\pm} \ln x_{\chi_j^\pm}) \\
 & \left. \left. + \frac{Q_f^2}{2x_z^2} \cdot (\varrho_{2,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm}) - x_{\chi_i^\pm} x_{\chi_j^\pm} \varrho_{0,1}(x_{\chi_i^\pm}, x_{\chi_j^\pm})) \right] \right\} \\
 & - 4Q_f^2 \Re(\xi_{ij}^L \xi_{ji}^R) s_w^2 (T_f^Z - Q_f s_w^2) (x_{\chi_i^\pm} x_{\chi_j^\pm})^{1/2} \frac{2 - \ln x_z + \ln x_R}{x_z^2} \Big\}, \\
 d_{l,\chi^\pm}^{ZZ} = & \frac{e^5 m_l}{(4\pi)^4 s_w^4 c_w^4 \Lambda^2} \cdot \Im(\xi_{ij}^L \xi_{ji}^R) (x_{\chi_i^\pm} x_{\chi_j^\pm})^{1/2} \left\{ Q_f s_w^2 (T_f^Z - Q_f s_w^2) \right. \\
 & \times \left( \frac{\partial^2}{\partial x_z \partial x_{\chi_j^\pm}} - \frac{\partial^2}{\partial x_z \partial x_{\chi_i^\pm}} \right) \left( \frac{\Phi(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) - \varphi_0(x_{\chi_i^\pm}, x_{\chi_j^\pm})}{x_z} \right) \\
 & \left. - \frac{1}{16} [(T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4] T_8(x_z, x_{\chi_i^\pm}, x_{\chi_j^\pm}) \right\}. \tag{41}
 \end{aligned}$$

In other words, the corrections from this sector to the MDM of lepton are decided by a linear combination of the real effective couplings  $|\xi_{ij}^L|^2 \pm |\xi_{ij}^R|^2$  and real parts of the effective couplings  $\xi_{ij}^L \xi_{ji}^R$ , and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings  $\xi_{ij}^L \xi_{ji}^R$ .

Because a real photon cannot be attached to the internal closed neutralino loop, the corresponding effective Lagrangian only contains the corrections to the lepton MDMs:

$$\begin{aligned}
 a_{l,\chi^0}^{ZZ} = & -\frac{e^4 Q_f m_l^2}{(4\pi)^4 s_w^4 c_w^4 \Lambda^2} \left\{ -\frac{1}{3} (|\eta_{ij}^L|^2 + |\eta_{ij}^R|^2) [(T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4] \right. \\
 & \times \left( T_5(x_z, x_{\chi_i^0}, x_{\chi_j^0}) + \frac{x_{\chi_i^0} + x_{\chi_j^0}}{x_z^2} \ln x_R \right) \\
 & \left. + \frac{1}{3} \Re(\eta_{ij}^L \eta_{ji}^R) [(T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4] (x_{\chi_i^0} x_{\chi_j^0})^{1/2} \left[ \frac{4}{x_z^2} \ln \frac{x_z}{x_R} - \frac{7}{x_z^2} \right] \right\}
 \end{aligned}$$



$$\begin{aligned}
& + \frac{1}{2x_z^2} (|\eta_{ij}^L|^2 + |\eta_{ij}^R|^2) Q_f s_w^2 (T_f^Z - Q_f s_w^2) \left[ \frac{x_z}{2} + (x_{\chi_i^0} \ln x_{\chi_i^0} + x_{\chi_j^0} \ln x_{\chi_j^0}) \right. \\
& - 2(x_{\chi_i^0} + x_{\chi_j^0}) \left( 2 - \ln \frac{x_z}{x_R} \right) - \mathcal{Q}_{2,1}(x_{\chi_i^0}, x_{\chi_j^0}) + x_{\chi_i^0} x_{\chi_j^0} \mathcal{Q}_{0,1}(x_{\chi_i^0}, x_{\chi_j^0}) \left. \right] \\
& - 4Q_f \Re(\eta_{ij}^L \eta_{ji}^R) s_w^2 (T_f^Z - Q_f s_w^2) (x_{\chi_i^0} x_{\chi_j^0})^{1/2} \frac{2 - \ln x_z + \ln x_R}{x_z^2} \Big\} \quad (42)
\end{aligned}$$

with

$$\begin{aligned}
\eta_{ij}^L &= \mathcal{N}_{i4}^\dagger \mathcal{N}_{4j}, \\
\eta_{ij}^R &= \mathcal{N}_{j3}^\dagger \mathcal{N}_{3i} \quad (i, j = 1, \dots, 4). \quad (43)
\end{aligned}$$

In order to get Eq. (42), we apply unitary property of the matrices  $\eta^{L,R}$ . The corrections from this sector to the MDM of lepton depend on a linear combination of the real effective couplings  $|\eta_{ij}^L|^2 \pm |\eta_{ij}^R|^2$  and real parts of the effective couplings  $\eta_{ij}^L \eta_{ji}^R$ , and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings  $\eta_{ij}^L \eta_{ji}^R$ .

We can also simplify Eqs. (41) and (42) in the limit  $x_{\chi_i^\pm}, x_{\chi_j^\pm}, x_{\chi_i^0}, x_{\chi_j^0} \gg x_z$  using the asymptotic expansion of  $\Phi(x, y, z)$ . The concrete expressions of  $T_4 \sim T_9$  can be found in [Appendix A](#).

## 2.6. The effective Lagrangian from the $WW$ sector

Similarly, the self energy of  $W$  gauge boson composed of a closed chargino–neutralino loop induces the ultraviolet divergence in the Wilson coefficients of effective Lagrangian. Accordingly, the unrenormalized  $W$  self energy is expressed as

$$\Sigma_{\mu\nu}^W(p) = \Lambda^2 A_0^W g_{\mu\nu} + \left( A_1^W + \frac{p^2}{\Lambda^2} A_2^W \right) (p^2 g_{\mu\nu} - p_\mu p_\nu) + \left( B_1^W + \frac{p^2}{\Lambda^2} B_2^W \right) p_\mu p_\nu. \quad (44)$$

The corresponding counter terms are given as

$$\Sigma_{\mu\nu}^{WC}(p) = -(\delta m_w^2 + m_w^2 \delta Z_w) g_{\mu\nu} - \delta Z_w (p^2 g_{\mu\nu} - p_\mu p_\nu). \quad (45)$$

The renormalized self energy is given by

$$\hat{\Sigma}_{\mu\nu}^W(p) = \Sigma_{\mu\nu}^W(p) + \Sigma_{\mu\nu}^{WC}(p). \quad (46)$$

For on-shell external gauge boson  $W^\pm$ , we have [12]

$$\begin{aligned}
\hat{\Sigma}_{\mu\nu}^W(p) \epsilon^\nu(p) \Big|_{p^2=m_w^2} &= 0, \\
\lim_{p^2 \rightarrow m_w^2} \frac{1}{p^2 - m_w^2} \hat{\Sigma}_{\mu\nu}^W(p) \epsilon^\nu(p) &= \epsilon_\mu(p), \quad (47)
\end{aligned}$$

where  $\epsilon(p)$  is the polarization vector of  $W$  gauge boson. Inserting Eqs. (44) and (45) into Eq. (47), we derive the counter terms for the  $W$  self energy as

$$\begin{aligned}
\delta Z_w &= A_1^W + \frac{m_w^2}{\Lambda^2} A_2^W = A_1^W + x_z A_2^W, \\
\delta m_w^2 &= A_0^W \Lambda^2 - m_w^2 \delta Z_w. \quad (48)
\end{aligned}$$

Differing from the analysis in the  $ZZ$  sector, we should derive the counter term for the vertex  $\gamma W^+ W^-$  here since the corresponding coupling is not zero at tree level. In the nonlinear  $R_\xi$

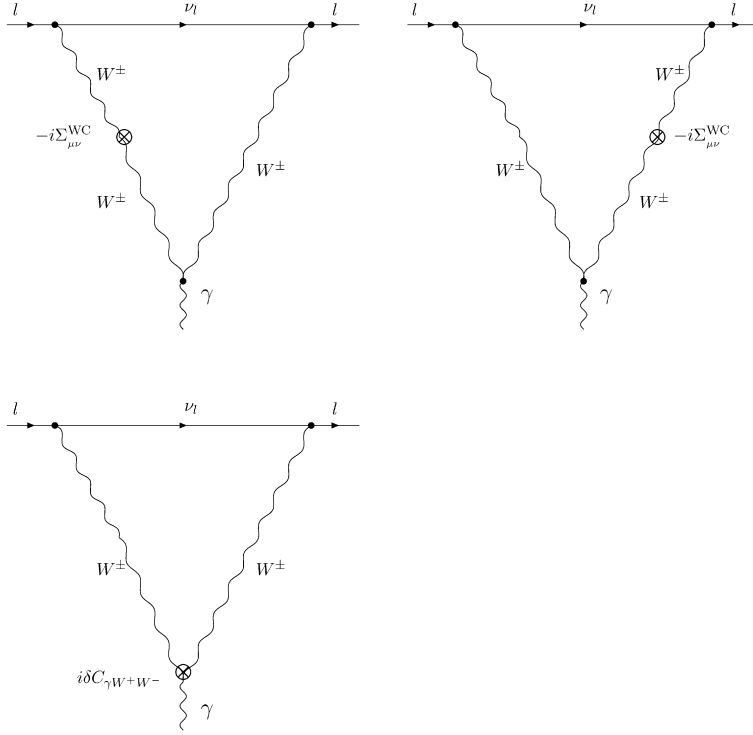


Fig. 3. The counter term diagram to cancel the ultraviolet caused by the self energy of  $W$  boson and electroweak radiative corrections to  $\gamma W^+ W^-$  vertex.

gauge with  $\xi = 1$ , the counter term for the vertex  $\gamma W^+ W^-$  is

$$i\delta C_{\gamma W^+ W^-} = ie \cdot \delta Z_W [g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu + g_{\rho\mu}(k_3 - k_1)_\nu], \quad (49)$$

where  $k_i$  ( $i = 1, 2, 3$ ) denote the injection momenta of  $W^\pm$  and photon, and  $\mu, \nu, \rho$  denote the corresponding Lorentz indices respectively.

We present the counter term diagrams to cancel the ultraviolet divergence contained in the bare effective Lagrangian from  $WW$  sector in Fig. 3, and we can verify that the sum of corresponding amplitude satisfies the Ward identity required by the QED gauge invariance obviously. Accordingly, the effective Lagrangian originating from the counter term diagrams can be written as

$$\begin{aligned} \delta \mathcal{L}_{WW}^C = & \frac{e^4}{(4\pi)^2 s_W^4 \Lambda^2 Q_f} (4\pi x_R)^{2\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{(1-\varepsilon)^2} \left\{ (\zeta_{ij}^{L*} \zeta_{ij}^L + \zeta_{ij}^{R*} \zeta_{ij}^R) \right. \\ & \times \left[ \frac{5}{24x_W^2} \left( -\frac{x_{\chi_i^0} + x_{\chi_j^\pm}}{\varepsilon} - \frac{x_{\chi_i^0} + x_{\chi_j^\pm}}{3} + \mathcal{Q}_{2,1}(x_{\chi_i^0}, x_{\chi_j^\pm}) \right) \right. \\ & \left. + (x_{\chi_i^0} + x_{\chi_j^\pm}) \ln x_R \right] + \frac{11}{36x_W} \left( \mathcal{O}_2^- + \mathcal{O}_3^- \right) + (\zeta_{ij}^{L*} \zeta_{ij}^R + \zeta_{ij}^{R*} \zeta_{ij}^L) (x_{\chi_i^0} x_{\chi_j^\pm})^{1/2} \\ & \left. \times \left[ \frac{5}{12x_W^2} \left( \frac{1}{\varepsilon} + \frac{5}{6} - \mathcal{Q}_{1,1}(x_{\chi_i^0}, x_{\chi_j^\pm}) - \ln x_R \right) \right] (\mathcal{O}_2^- + \mathcal{O}_3^-) \right\} + \dots \quad (50) \end{aligned}$$

Finally, we get the renormalized effective Lagrangian from the  $WW$  sector:

$$\begin{aligned}
 \mathcal{L}_{WW} = & -\frac{e^4}{48(4\pi)^2 s_w^4 Q_f \Lambda^2} (\zeta_{ij}^{L*} \zeta_{ij}^L + \zeta_{ij}^{R*} \zeta_{ij}^R) \left[ T_{10}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \right. \\
 & + \frac{10}{x_w^2} (x_{\chi_i^0} + x_{\chi_j^\pm}) \ln x_R \left. \right] (\mathcal{O}_2^- + \mathcal{O}_3^-) \\
 & - \frac{e^4}{16(4\pi)^2 s_w^4 Q_f \Lambda^2} (\zeta_{ij}^{L*} \zeta_{ij}^L - \zeta_{ij}^{R*} \zeta_{ij}^R) T_{11}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) (\mathcal{O}_2^- + \mathcal{O}_3^-) \\
 & - \frac{e^4 (x_{\chi_i^0} x_{\chi_j^\pm})^{1/2}}{48(4\pi)^2 s_w^4 Q_f \Lambda^2} (\zeta_{ij}^{L*} \zeta_{ij}^R + \zeta_{ij}^{R*} \zeta_{ij}^L) \left[ T_{12}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \right. \\
 & - \frac{20}{x_w^2} \ln x_R \left. \right] (\mathcal{O}_2^- + \mathcal{O}_3^-) \\
 & - \frac{e^4 (x_{\chi_i^0} x_{\chi_j^\pm})^{1/2}}{16(4\pi)^2 s_w^4 Q_f \Lambda^2} (\zeta_{ij}^{R*} \zeta_{ij}^L - \zeta_{ij}^{L*} \zeta_{ij}^R) T_{13}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) (\mathcal{O}_2^- - \mathcal{O}_3^-). \quad (51)
 \end{aligned}$$

Correspondingly, the resulted lepton MDMs and EDMs are respectively formulated as

$$\begin{aligned}
 a_l^{WW} = & -\frac{e^4 m_l^2}{12(4\pi)^4 s_w^4 \Lambda^2} (|\zeta_{ij}^L|^2 + |\zeta_{ij}^R|^2) \left[ T_{10}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \right. \\
 & + \frac{10}{x_w^2} (x_{\chi_i^0} + x_{\chi_j^\pm}) \ln x_R - \frac{32}{x_w} \ln x_R \left. \right] \\
 & - \frac{e^4 m_l^2}{4(4\pi)^4 s_w^4 \Lambda^2} (|\zeta_{ij}^L|^2 - |\zeta_{ij}^R|^2) T_{11}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) \\
 & - \frac{e^4 m_l^2 (x_{\chi_i^0} x_{\chi_j^\pm})^{1/2}}{6(4\pi)^4 s_w^4 \Lambda^2} \Re(\zeta_{ij}^{R*} \zeta_{ij}^L) \left[ T_{12}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}) - \frac{20}{x_w^2} \ln x_R \right], \\
 d_l^{WW} = & -\frac{e^5 m_l (x_{\chi_i^0} x_{\chi_j^\pm})^{1/2}}{4(4\pi)^4 s_w^4 \Lambda^2} \Im(\zeta_{ij}^{R*} \zeta_{ij}^L) T_{13}(x_w, x_{\chi_i^0}, x_{\chi_j^\pm}). \quad (52)
 \end{aligned}$$

In a similar way, the corrections from this sector to the MDM of lepton depend on a linear combination of the real effective couplings  $|\zeta_{ij}^L|^2 \pm |\zeta_{ij}^R|^2$  and real parts of the effective couplings  $\zeta_{ij}^L \zeta_{ij}^{R*}$ , and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings  $\zeta_{ij}^L \zeta_{ij}^{R*}$ .

### 3. Numerical results and discussion

With the theoretical formulae derived in previous section, we numerically analyze the dependence of the muon MDM and EDM on the supersymmetric parameters in the CP-violating scenario here. In particular, we will present the dependence of the muon MDM and EDM on the supersymmetric CP phases in some detail. In order to make the theoretical predictions on the electron and neutron EDMs satisfying the present experimental constraints, we adopt the cancellation mechanism among the different contributions to the fermion EDMs [20]. Within three standard error deviations, the present experimental data can tolerate new physics correction to the muon MDM as  $2.6 \times 10^{-10} < \Delta a_\mu < 58.4 \times 10^{-10}$ . Since the neutralinos  $\chi_i^0$  ( $i = 1, 2, 3, 4$ )

and charginos  $\chi_i^\pm$  ( $i = 1, 2$ ) appear as the internal intermediate particles in the two-loop diagrams which are investigated in this work, the corrections of these diagrams will be suppressed strongly when the masses of neutralinos and charginos are much higher than the electroweak scale [7]. To investigate if those diagrams can result in concrete corrections to the muon MDM and EDM, we choose a suitable supersymmetric parameter region where the masses of neutralinos and charginos are lying in the range  $M_\chi < 600$  GeV.

The MSSM Lagrangian contains several sources for CP violating phases: the phases of the  $\mu$ -parameter in the superpotential and the corresponding bilinear coupling of the soft breaking terms, three phases of the gaugino masses, and the phases of the scalar fermion Yukawa couplings in the soft Lagrangian. As we do not consider the spontaneous CP violation in this work, the CP phase of soft bilinear coupling vanishes due to the neutral Higgs tadpole conditions. Additional, the CP violation would cause changes to the neutral-Higgs-quark coupling, the neutral Higgs-gauge-boson coupling and the self-coupling of Higgs boson. A direct result of above facts is that no absolute limits can be set for the Higgs bosons masses from the present combined LEP data [23]. For security, we take the lower bound on the mass of the lightest Higgs boson as  $m_{h_1} \geq 60$  GeV [14] in the numerical analysis. In order to obtain the mixing matrix of neutral Higgs in CP violating MSSM, we include the subroutine *fillhiggs.f* from the Package CPsuperH [24] in our numerical code. Furthermore, we take the pole mass of top quark  $m_t(\text{pole}) = 175$  GeV, the pole mass of charged Higgs  $m_{H^\pm}(\text{pole}) = 300$  GeV, the running masses  $m_b(m_t) = 3$  GeV,  $m_\tau(m_t) = 1.77$  GeV, the mass parameters of scalar fermions in soft terms as  $m_{\tilde{U}_3} = m_{\tilde{D}_3} = m_{\tilde{E}_3} = m_{\tilde{Q}_3} = m_{\tilde{L}_3} = 500$  GeV, the Yukawa couplings of scalar fermions as  $|A_t| = |A_b| = |A_\tau| = 1$  TeV and  $\phi_{A_t} = \phi_{A_b} = \phi_{A_\tau} = \pi/2$ . Fixing above parameters and assuming  $\tan\beta \geq 3$ , we find that the mass of the lightest neutral Higgs is well above 115 GeV by scanning the parameter space of CP violating MSSM. In other words, one no longer worries about the constraint from Higgs sector with the above assumptions on the parameter space of CP violating MSSM. With no loss of generality, we also take the supersymmetric parameters  $|m_1| = |m_2| = 500$  GeV and  $m_{\tilde{E}_2} = m_{\tilde{L}_2} = A_\mu/2 = 500$  GeV in this work.

Taking  $|\mu_H| = 200$  GeV,  $\phi_{m_2} = \phi_{\mu_H} = 0$  and  $\tan\beta = 10, 50$ , we plot the muon MDM  $a_\mu$  and EDM  $d_\mu$  versus the CP phase  $\phi_{m_1}$  in Fig. 4. As  $\tan\beta = 10$ , the one-loop supersymmetric correction to the muon MDM (solid-line in Fig. 4(a)) reaches  $7 \times 10^{-10}$  and can account for the deviation between the SM prediction and experimental data. Comparing with one-loop supersymmetric contribution, two-loop contribution depends on the supersymmetric parameters in a different manner. Including the two-loop corrections, the supersymmetric contribution to the muon MDM  $a_\mu$  is modified about 10%. Since the gaugino mass  $m_1$  affects the theoretical prediction only through the mixing matrix of neutralinos, the muon MDM  $a_\mu$  varies with the CP phase  $\phi_{m_1}$  (solid line for one-loop result and dot line for the result including two-loop corrections in Fig. 4(a)) very mildly. Meanwhile the supersymmetric contribution to the muon EDM including two-loop corrections at the largest CP violation  $\phi_{m_1} = \pi/2$  is still below  $10^{-24}$  e cm (dot line Fig. 4(b)), and it is very difficult to observe the muon EDM of this level in next generation experiments with precision  $10^{-24}$  e cm [17]. As  $\tan\beta = 50$ , one-loop supersymmetric correction to the muon MDM  $a_\mu$  exceeds  $35 \times 10^{-10}$  (dash line in Fig. 4(a)), and can ameliorate easily the discrepancy between the SM prediction and experiment. Because the dominant two-loop supersymmetric corrections originating from the  $\gamma h_k$ ,  $W^\pm H^\mp$  sectors are enhanced by large  $\tan\beta$ , the relative modification from two-loop supersymmetric corrections to one-loop result is 15% roughly (dash-dot line in Fig. 4(a)). As for the muon EDM  $d_\mu$ , one-loop supersymmetric result together with two-loop supersymmetric corrections are all enhanced by large  $\tan\beta$ . The contri-

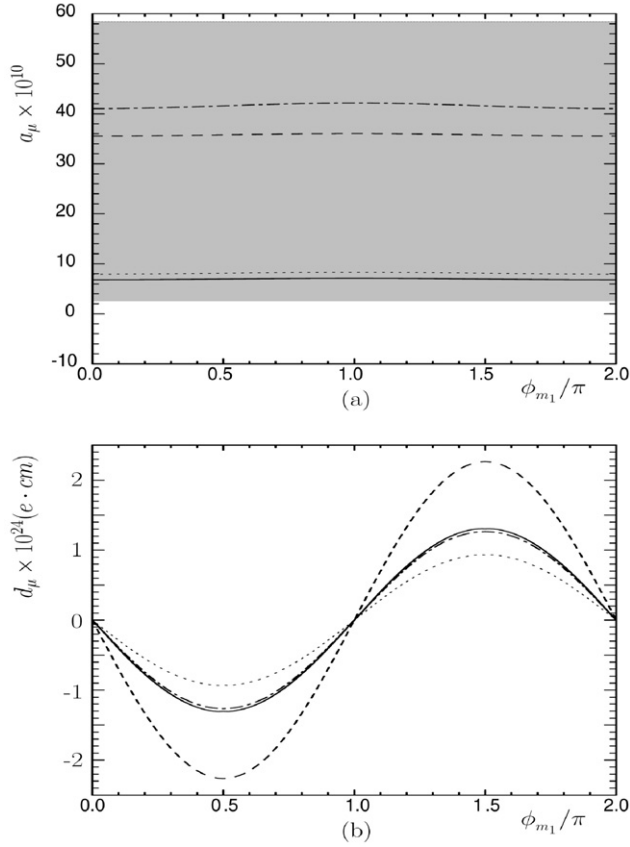


Fig. 4. The supersymmetric corrections to the muon MDM  $a_\mu$  and EDM  $d_\mu$  vary with the CP violating phase  $\phi_{m_1}$  when  $|\mu_H| = 200$  GeV,  $\phi_{m_2} = \phi_{\mu_H} = 0$  and  $\tan\beta = 10, 50$ , where the solid lines stand for the one-loop corrections with  $\tan\beta = 10$ , the dot lines stand for the results including two-loop supersymmetric corrections with  $\tan\beta = 10$ ; the dash lines stand for the one-loop corrections with  $\tan\beta = 50$ , the dash-dot lines stand for the results including two-loop supersymmetric corrections with  $\tan\beta = 50$ . The gray band in diagram (a) is the region allowed by the  $g-2$  experimental data within 3 standard errors.

bution including two-loop supersymmetric corrections is well above  $10^{-24} e \text{ cm}$  at the largest CP violation  $\phi_{m_1} = \pi/2$ , and it is hopeful to detect the muon EDM  $d_\mu$  of this level in the near future.

Taking  $|\mu_H| = 200$  GeV,  $\phi_{m_1} = \phi_{\mu_H} = 0$  and  $\tan\beta = 10, 50$ , we plot the muon MDM  $a_\mu$  and EDM  $d_\mu$  versus the CP phase  $\phi_{m_2}$  in Fig. 5. As  $\tan\beta = 10$ , the one-loop supersymmetric correction to the muon MDM (solid-line in Fig. 5(a)) always lies in the range  $|a_\mu| < 8 \times 10^{-10}$  varying with the CP phase  $\phi_{m_2}$ . The relative modification from the two-loop supersymmetric corrections to the one-loop prediction is below 5% when  $\tan\beta = 10$ . Since the gaugino mass  $m_2$  affects the theoretical prediction through the mixing matrices of neutralinos and charginos simultaneously, the muon MDM  $a_\mu$  depends on the CP phase  $\phi_{m_2}$  (solid line for one-loop result and dot line for the result including two-loop corrections in Fig. 5(a)) strongly. Meanwhile the supersymmetric contribution to the muon EDM including two-loop corrections at the largest CP violation  $\phi_{m_2} = \pi/2$  is about  $10^{-23} e \text{ cm}$  (dot line Fig. 5(b)) which can be observed in next generation experiments with precision  $10^{-24} e \text{ cm}$  [17]. When  $\tan\beta = 50$ , one-loop supersymmetric correction

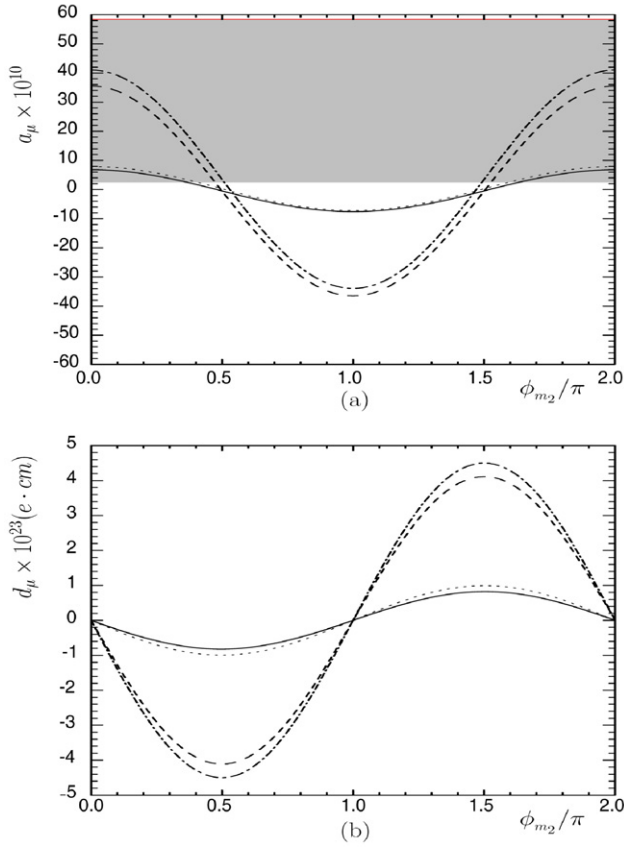


Fig. 5. The supersymmetric corrections to the muon MDM  $a_\mu$  and EDM  $d_\mu$  vary with the CP violating phase  $\phi_{m_2}$  when  $|\mu_H| = 200$  GeV,  $\phi_{m_1} = \phi_{\mu_H} = 0$  and  $\tan\beta = 10, 50$ , where the solid lines stand for the one-loop corrections with  $\tan\beta = 10$ , the dot lines stand for the results including two-loop supersymmetric corrections with  $\tan = 10$ ; the dash lines stand for the one-loop corrections with  $\tan\beta = 50$ , the dash-dot lines stand for the results including two-loop supersymmetric corrections with  $\tan = 50$ . The gray band in diagram (a) is the region allowed by the  $g - 2$  experimental data within 3 standard errors.

to the muon MDM  $a_\mu$  is enhanced drastically. Because the dominant two-loop supersymmetric corrections originating from the  $\gamma h_k$ ,  $W^\pm H^\mp$  sectors are also enhanced by large  $\tan\beta$ , the relative modification from two-loop supersymmetric corrections to one-loop result is 15% roughly (dash-dot line in Fig. 5(a)). As for the muon EDM  $d_\mu$ , one-loop supersymmetric result together with two-loop supersymmetric corrections are all enhanced by large  $\tan\beta$ . The contribution including two-loop supersymmetric corrections at the largest CP violation  $\phi_{m_2} = \pi/2$  is about  $4 \times 10^{-23} e \text{ cm}$  which can be detected easily in next generation experiments.

Taking  $|\mu_H| = 200$  GeV,  $\phi_{m_1} = \phi_{m_2} = 0$  and  $\tan\beta = 10, 50$ , we plot the muon MDM  $a_\mu$  and EDM  $d_\mu$  versus the CP phase  $\phi_{\mu_H}$  in Fig. 6. As  $\tan\beta = 10$ , the one-loop supersymmetric correction to the muon MDM (solid-line in Fig. 6(a)) always lies in the range  $|a_\mu| < 8 \times 10^{-10}$  varying with the CP phase  $\phi_{\mu_H}$ . The relative modification from the two-loop supersymmetric corrections to the one-loop prediction is below 5% when  $\tan\beta = 10$ . Since the  $\mu$  parameter  $\mu_H$  affects the theoretical prediction through the mixing matrices of neutralinos and charginos si-

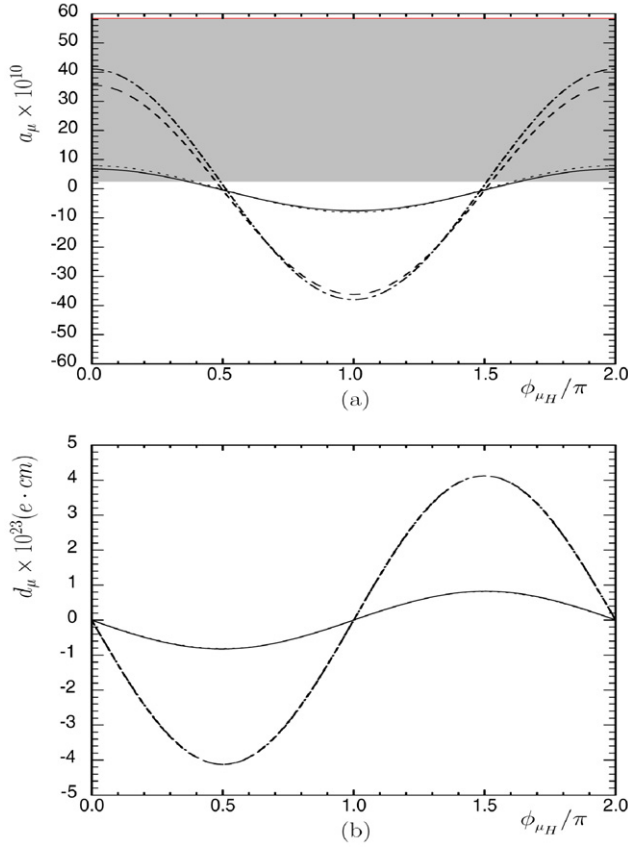


Fig. 6. The supersymmetric corrections to the muon MDM  $a_\mu$  and EDM  $d_\mu$  vary with the CP violating phase  $\phi_{\mu H}$  when  $|\mu_H| = 200$  GeV,  $\phi_{m_1} = \phi_{m_2} = 0$  and  $\tan\beta = 10, 50$ , where the solid lines stand for the one-loop corrections with  $\tan\beta = 10$ , the dot lines stand for the results including two-loop supersymmetric corrections with  $\tan = 10$ ; the dash lines stand for the one-loop corrections with  $\tan\beta = 50$ , the dash-dot lines stand for the results including two-loop supersymmetric corrections with  $\tan = 50$ . The gray band in diagram (a) is the region allowed by the  $g-2$  experimental data within 3 standard errors.

multaneously, the muon MDM  $a_\mu$  varies with the CP phase  $\phi_{\mu H}$  (solid line for one-loop result and dot line for the result including two-loop corrections in Fig. 6(a)) drastically. Meanwhile the supersymmetric contribution to the muon EDM including two-loop corrections at the largest CP violation  $\phi_{\mu H} = \pi/2$  is below  $10^{-23} e \text{ cm}$  (dot line Fig. 6(b)). Because the dominant two-loop supersymmetric corrections originating from the  $\gamma h_k$ ,  $W^\pm H^\mp$  sectors are enhanced by large  $\tan\beta$ , the relative modification from two-loop supersymmetric corrections to one-loop result is 15% roughly (dash-dot line in Fig. 6(a)) at CP conservation when  $\tan\beta = 50$ . One-loop supersymmetric correction to the muon EDM  $d_\mu$  is enhanced by large  $\tan\beta$ . Comparing with one-loop contribution, two-loop corrections are negligible. The contribution including two-loop supersymmetric corrections is about  $4 \times 10^{-23} e \text{ cm}$ , which can be detected in the near future [17].

Taking  $\tan\beta = 20, 50$  and  $\phi_{m_1} = \phi_{m_2} = \phi_{\mu H} = 0$ , we plot the muon MDM  $a_\mu$  versus the  $\mu$ -parameter  $\mu_H$  in Fig. 7. The gray band is the region allowed by present experimental data within 3 standard errors. Because the supersymmetric corrections to the muon MDM  $a_\mu$  are negative for

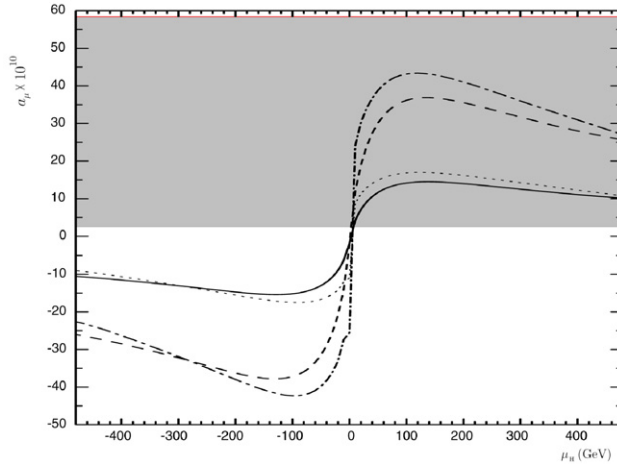


Fig. 7. The supersymmetric corrections to the muon MDM  $a_\mu$  vary with the  $\mu$ -parameter  $\mu_H$  when  $\phi_{m_1} = \phi_{m_2} = \phi_{\mu_H} = 0$  and  $\tan\beta = 20, 50$ , where the solid lines stand for the one-loop corrections with  $\tan\beta = 20$ , the dot lines stand for the results including two-loop supersymmetric corrections with  $\tan\beta = 20$ ; the dash lines stand for the one-loop corrections with  $\tan\beta = 50$ , the dash-dot lines stand for the results including two-loop supersymmetric corrections with  $\tan\beta = 50$ . The gray band is the region allowed by the  $g-2$  experimental data within 3 standard errors.

$\mu_H \leq 0$ , the corresponding parameter space is already ruled out by the present  $g-2$  experimental data. Comparing with the one-loop supersymmetric results (solid line for  $\tan\beta = 20$  and dash line for  $\tan\beta = 50$  respectively), the contributions including two-loop supersymmetric corrections are enhanced about 15% when  $\mu_H = 150$  GeV. Along with the increasing of  $\mu_H$ , the two-loop corrections become more and more trivial.

#### 4. Conclusions

In this work, we analyzed the two-loop supersymmetric corrections to the muon MDM and EDM by the effective Lagrangian method in the CP violating MSSM. In the concrete calculation, we keep all dimension 6 operators. The ultraviolet divergence caused by the divergent subdiagrams is removed in the on-shell renormalization schemes. After applying the equations of motion to the external leptons, we derive the muon MDM and EDM. Numerically, we analyze the dependence of the muon MDM  $a_\mu$  and EDM  $d_\mu$  on supersymmetric CP violating phases. As discussed above,  $a_\mu$  is decided by real parts of the effective couplings, and  $d_\mu$  is decided by imaginary parts of the effective couplings after the heavy freedoms are integrated out. Adopting our assumptions on parameter space of the MSSM and choosing  $\tan\beta = 50$ , we find that the correction from those two-loop diagrams to  $a_\mu$  is  $4 \times 10^{-10}$  roughly for the case of CP conservation, which lies in the order of present experimental precision in magnitude. In other words, the present experimental data put a very restrictive bound on the real parts of those effective couplings. Additional, the contribution to  $d_\mu$  from this sector is sizable enough to be experimentally detected with the experimental precision of near future.

#### Acknowledgements

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## Appendix A. The functions

We list the tedious expressions of the functions adopted in the text

$$\varrho_{i,j}(x, y) = \frac{x^i \ln^j x - y^i \ln^j y}{x - y},$$

$$\Omega_n(x, y; u, v) = \frac{x^n \Phi(x, u, v) - y^n \Phi(y, u, v)}{x - y},$$

$$\begin{aligned} T_1(x_1, x_2, x_3) &= \frac{1}{x_1} \left\{ -4(2 + \ln x_2)(\ln x_1 - 1) - \frac{\partial}{\partial x_3} \left[ \left( 1 + 2 \frac{x_2 - x_3}{x_1} \right) \Phi \right] (x_1, x_2, x_3) \right. \\ &\quad \left. + \frac{\partial}{\partial x_3} \left[ \left( 1 + 2 \frac{x_2 - x_3}{x_1} \right) \varphi_0 + 2(x_2 - x_3) \varphi_1 \right] (x_2, x_3) \right\}, \end{aligned}$$

$$T_2(x_1, x_2, x_3) = \frac{1}{x_1} \left[ \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right],$$

$$\begin{aligned} T_3(x_1, x_2, x_3) &= -\frac{2}{x_1} (2 + \ln x_3) + \frac{2}{x_1} \frac{\partial^2}{\partial x_3^2} (x_3 \Phi)(x_1, x_2, x_3) \\ &\quad - \frac{2}{x_1} \frac{\partial^2}{\partial x_3^2} (x_3 \varphi_0)(x_2, x_3) - \frac{4}{x_1} \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) + \frac{4}{x_1} \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \\ &\quad + \frac{\partial^2}{\partial x_1 \partial x_3} \left( \frac{x_2 - x_3}{x_1} \varphi_0 \right) (x_2, x_3) + \frac{\partial^2}{\partial x_1 \partial x_3} \left[ \left( 1 - \frac{x_2 - x_3}{x_1} \right) \Phi \right] (x_1, x_2, x_3), \end{aligned}$$

$$\begin{aligned} T_4(x_1, x_2, x_3) &= \frac{2}{x_1} \ln x_3 - \frac{2}{x_1^2} (x_2 - x_2 \ln x_2 - x_3 + x_3 \ln x_3) \\ &\quad - \frac{\partial^3}{\partial x_1 \partial x_3^2} \left[ \frac{x_2 x_3 - x_3^2}{x_1} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) \right] \\ &\quad + \frac{1}{2} \frac{\partial^3}{\partial x_1^2 \partial x_3} [(x_2 - 3x_3 - x_1) \Phi(x_1, x_2, x_3)] \\ &\quad - \frac{1}{2} \frac{\partial^2}{\partial x_1 \partial x_3} \left[ \Phi(x_1, x_2, x_3) - \frac{5}{x_1} (x_2 - x_3) (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) \right] \\ &\quad - \frac{\partial^2}{\partial x_1^2} \left[ \frac{x_2 - x_3}{x_1} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) + 2\Phi(x_1, x_2, x_3) \right], \end{aligned}$$

$$\begin{aligned} T_5(x_1, x_2, x_3) &= \frac{5}{12x_1} + \left( \frac{5}{12x_1^2} + \frac{\ln x_1}{3x_1^2} + \frac{\ln x_R}{x_1^2} \right) (x_2 + x_3) \\ &\quad + \left( \frac{7}{6x_1^2} + \frac{2}{3x_1^2} \ln x_1 \right) (x_2 \ln x_2 + x_3 \ln x_3) \\ &\quad + \left( \frac{2}{3x_1^3} - \frac{4}{3x_1^3} \ln x_1 \right) (x_2 - x_3)^2 (1 + \varrho_{1,1}(x_2, x_3)) \end{aligned}$$

$$\begin{aligned}
& + \frac{23}{6x_1^2}(x_2 + x_3)(1 + \varrho_{1,1}(x_2, x_3)) - \frac{5\varrho_{2,1}(x_2, x_3)}{x_1^2} \\
& - \frac{1}{3x_1^2} \left( 1 - \frac{2(x_2 + x_3)}{x_1} \right) (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) \\
& + \frac{1}{3x_1} \left( \frac{x_2 + x_3}{x_1} - \frac{2(x_2 - x_3)^2}{x_1^2} \right) \varphi_1(x_2, x_3) \\
& + \frac{1}{3x_1} \left( 1 - \frac{3(x_2 + x_3)}{x_1} + \frac{2(x_2 - x_3)^2}{x_1^2} \right) \frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3) \\
& - \frac{1}{3} \left( 1 - \frac{2(x_2 + x_3)}{x_1} + \frac{(x_2 - x_3)^2}{x_1^2} \right) \frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3) \\
& - \frac{(x_2 - x_3)^2}{3x_1^2} \varphi_2(x_2, x_3),
\end{aligned}$$

$$\begin{aligned}
& T_6(x_1, x_2, x_3) \\
& = -\frac{1}{x_1^2} \left( \varphi_0 - (x_2 - x_3) \frac{\partial \varphi_0}{\partial x_3} \right) (x_2, x_3) + \left[ 2x_3 \frac{\partial^3 \Phi}{\partial x_1 \partial x_3^2} + \frac{\partial^2 \Phi}{\partial x_1^2} \right. \\
& \quad + (x_1 - x_2 + x_3) \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3} + \frac{\Phi}{x_1^2} - \frac{x_2 - x_3}{x_1^2} \frac{\partial \Phi}{\partial x_3} - \frac{1}{x_1} \frac{\partial \Phi}{\partial x_1} \\
& \quad \left. + \left( 1 + \frac{x_2 - x_3}{x_1} \right) \frac{\partial^2 \Phi}{\partial x_1 \partial x_3} \right] (x_1, x_2, x_3),
\end{aligned}$$

$$\begin{aligned}
& T_7(x_1, x_2, x_3) \\
& = -2 \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3} (x_1, x_2, x_3) + \frac{2}{x_1 x_3} - \frac{2}{x_1^2} (\ln x_2 - \ln x_3) \\
& \quad + \left( \frac{\partial^3}{\partial x_1^2 \partial x_3} - \frac{\partial^3}{\partial x_1 \partial x_3^2} + \frac{\partial^3}{\partial x_1^2 \partial x_2} + \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3} \right) \left[ \Phi(x_1, x_2, x_3) \right. \\
& \quad \left. - \frac{x_2 - x_3}{x_1} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) \right],
\end{aligned}$$

$$\begin{aligned}
& T_8(x_1, x_2, x_3) \\
& = -4 \left( \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3} + \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_2} \right) (x_1, x_2, x_3) + \frac{4}{x_1 x_3} + \frac{2}{x_1^2} (2 + \ln x_2) \\
& \quad + \left( 2 \frac{\partial^3}{\partial x_1 \partial x_3^2} + \frac{\partial^3}{\partial x_1^2 \partial x_2} \right) \left[ \frac{x_2 - x_3}{x_1} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) - \Phi(x_1, x_2, x_3) \right],
\end{aligned}$$

$$\begin{aligned}
& T_9(x_1, x_2, x_3) \\
& = \frac{2}{x_1} \ln x_3 - \frac{4x_3}{x_1^2} \left( \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right) \\
& \quad + \frac{\partial^2}{\partial x_1 \partial x_3} \left( (x_2 - x_3) \frac{\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)}{x_1} - \Phi(x_1, x_2, x_3) \right) \\
& \quad + \frac{4}{x_1} \left( \frac{\partial \Phi}{\partial x_3} - \frac{\partial \Phi}{\partial x_1} \right) (x_1, x_2, x_3) + \frac{4x_3}{x_1} \frac{\partial^2 \Phi}{\partial x_1 \partial x_3} (x_1, x_2, x_3),
\end{aligned}$$

$$\begin{aligned}
& T_{10}(x_1, x_2, x_3) \\
&= \frac{26}{x_1} + \frac{17x_2}{x_1^2} + \frac{29x_3}{x_1^2} + \frac{10}{x_1^2} \varrho_{2,1}(x_2, x_3) - \frac{16(x_2 - x_3)^2}{x_1^3} - \frac{10(x_2 + x_3)}{x_1^2} \ln x_1 \\
&\quad - \frac{6 \ln x_3}{x_1} + \left[ 14 - \frac{16(x_2 - x_3)}{x_1} \right] \frac{x_2 \ln x_2}{x_1^2} + \left[ -4 + \frac{16(x_2 - x_3)}{x_1} \right] \frac{x_3 \ln x_3}{x_1^2} \\
&\quad + [(x_2 - x_3)^2 - x_1^2] \frac{\partial^4 \Phi}{\partial x_1^4}(x_1, x_2, x_3) \\
&\quad + \left[ -5x_1 + 6x_2 + \frac{3(x_2 - x_3)^2}{x_1} \right] \frac{\partial^3 \Phi}{\partial x_1^3}(x_1, x_2, x_3) \\
&\quad + \left[ -\frac{9(x_2 - x_3)^2}{x_1^2} + \frac{6x_2}{x_1} + \frac{3x_3}{x_1} \right] \frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3) \\
&\quad + \left[ -\frac{12x_2}{x_1^2} - \frac{6x_3}{x_1^2} + \frac{18(x_2 - x_3)^2}{x_1^3} \right] \frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3) \\
&\quad + \left[ \frac{12x_2}{x_1^3} + \frac{6x_3}{x_1^3} - \frac{18(x_2 - x_3)^2}{x_1^4} \right] (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) \\
&\quad + \frac{2x_3^2(x_2 - x_3)}{x_1^2} \left[ \frac{\partial^3 \Phi}{\partial x_3^3}(x_1, x_2, x_3) - \frac{\partial^3 \varphi_0}{\partial x_3^3}(x_2, x_3) \right] \\
&\quad + \left[ \frac{3x_\alpha x_\beta}{x_1^2} - \frac{9x_\beta^2}{x_1^2} \right] \left[ \frac{\partial^2 \Phi}{\partial x_3^2}(x_1, x_2, x_3) - \frac{\partial^2 \varphi_0}{\partial x_3^2}(x_2, x_3) \right] \\
&\quad - \left[ \frac{3x_\alpha}{x_1^2} + \frac{9x_\beta}{x_1^2} + \frac{18x_3(x_2 - x_3)}{x_1^3} \right] \left[ \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right] \\
&\quad - 6x_3(x_2 - x_3 + x_1) \frac{\partial^4 \Phi}{\partial x_1^3 \partial x_3}(x_1, x_2, x_3) + 6x_3(x_2 + x_3 - x_1) \frac{\partial^4 \Phi}{\partial x_1^2 \partial x_3^2}(x_1, x_2, x_3) \\
&\quad - 2x_3^2 \left( 1 + \frac{x_2 - x_3}{x_1} \right) \frac{\partial^4 \Phi}{\partial x_1 \partial x_3^3}(x_1, x_2, x_3) \\
&\quad + \left[ 3x_1 - 3x_2 - 18x_3 - \frac{9x_3(x_2 - x_3)}{x_1} \right] \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3}(x_1, x_2, x_3) \\
&\quad + \left[ -21x_3 - \frac{3x_2 x_3}{x_1} + \frac{9x_3^2}{x_1} \right] \frac{\partial^3 \Phi}{\partial x_1 \partial x_3^2}(x_1, x_2, x_3) \\
&\quad - \left[ 6 - \frac{12x_2}{x_1} + \frac{6x_3}{x_1} - \frac{18x_3(x_2 - x_3)}{x_1^2} \right] \frac{\partial^2 \Phi}{\partial x_1 \partial x_3}(x_1, x_2, x_3), \\
& T_{11}(x_1, x_2, x_3) \\
&= \frac{2 \ln x_3}{x_1} - \frac{4(x_2 - x_3)}{x_1^2} - \frac{4(x_2 \ln x_2 - x_3 \ln x_3)}{x_1^2} \\
&\quad - \frac{4(x_2 - x_3)}{x_1^3} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) + \frac{4(x_2 - x_3)}{x_1^2} \frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3)
\end{aligned}$$

$$\begin{aligned}
& - \left( 1 + \frac{2(x_2 - x_3)}{x_1} \right) \frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3) - \frac{2x_3}{x_1^2} \left( \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right) \\
& + \frac{x_3(x_2 - x_3)}{x_1^2} \left( \frac{\partial^2 \Phi}{\partial x_3^2}(x_1, x_2, x_3) - \frac{\partial^2 \varphi_0}{\partial x_3^2}(x_2, x_3) \right) \\
& - 2 \frac{\partial^2 \Phi}{\partial x_1 \partial x_3}(x_1, x_2, x_3) - x_3 \left( 1 + \frac{x_2 - x_3}{x_1} \right) \frac{\partial^3 \Phi}{\partial x_1 \partial x_3^2}(x_1, x_2, x_3) \\
& + (x_2 + x_3 - x_1) \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3}(x_1, x_2, x_3),
\end{aligned}$$

$$T_{12}(x_1, x_2, x_3)$$

$$\begin{aligned}
& = -\frac{52}{x_1^2} + \frac{4}{x_1 x_3} + \frac{20}{x_1^2} (\ln x_1 - \varrho_{1,1}(x_2, x_3)) - \frac{18 \ln x_3}{x_1^2} \\
& - \frac{12}{x_1^3} (\Phi(x_1, x_2, x_3) - \varphi_0(x_2, x_3)) + \frac{12}{x_1^2} \frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3) \\
& - \frac{6}{x_1} \frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3) - \left( 17 \frac{\partial^3 \Phi}{\partial x_1^3} + 2x_1 \frac{\partial^4 \Phi}{\partial x_1^4} \right)(x_1, x_2, x_3) \\
& + \frac{6}{x_1^2} \left( 1 + \frac{2(x_2 - x_3)}{x_1} \right) \left( \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right) \\
& - \frac{3(x_2 - 2x_3)}{x_1^2} \left( \frac{\partial^2 \Phi}{\partial x_3^2}(x_1, x_2, x_3) - \frac{\partial^2 \varphi_0}{\partial x_3^2}(x_2, x_3) \right) \\
& - \frac{x_3(x_2 - x_3)}{x_1^2} \left( \frac{\partial^3 \Phi}{\partial x_3^3}(x_1, x_2, x_3) - \frac{\partial^3 \varphi_0}{\partial x_3^3}(x_2, x_3) \right) \\
& - x_3 \left( 1 - \frac{x_2 - x_3}{x_1} \right) \frac{\partial^4 \Phi}{\partial x_1 \partial x_3^3}(x_1, x_2, x_3) - \frac{6}{x_1} \left( 1 + \frac{2(x_2 - x_3)}{x_1} \right) \frac{\partial^2 \Phi}{\partial x_1 \partial x_3}(x_1, x_2, x_3) \\
& - \left[ 3 \left( 1 - \frac{x_2 - 2x_3}{x_1} \right) \frac{\partial^3 \Phi}{\partial x_1 \partial x_3^2} + 6 \left( 2 - \frac{x_2 - x_3}{x_1} \right) \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3} \right](x_1, x_2, x_3) \\
& + 3(x_2 - x_3 - x_1) \frac{\partial^4 \Phi}{\partial x_1^3 \partial x_3}(x_1, x_2, x_3) - 6 \frac{\partial^4 \Phi}{\partial x_1^2 \partial x_3^2}(x_1, x_2, x_3),
\end{aligned}$$

$$T_{13}(x_1, x_2, x_3)$$

$$\begin{aligned}
& = \frac{1}{x_1 x_3} + \frac{2}{x_1^2} \left( \frac{\partial \Phi}{\partial x_3}(x_1, x_2, x_3) - \frac{\partial \varphi_0}{\partial x_3}(x_2, x_3) \right) - \frac{2}{x_1} \frac{\partial^2 \Phi}{\partial x_1 \partial x_3}(x_1, x_2, x_3) \\
& - \frac{x_2 - x_3}{x_1^2} \left( \frac{\partial^2 \Phi}{\partial x_3^2}(x_1, x_2, x_3) - \frac{\partial^2 \varphi_0}{\partial x_3^2}(x_2, x_3) \right) \\
& - \left( 1 - \frac{x_2 - x_3}{x_1} \right) \frac{\partial^3 \Phi}{\partial x_1 \partial x_3^2}(x_1, x_2, x_3) - 2 \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_3}(x_1, x_2, x_3),
\end{aligned}$$

$$F_1(x_1, x_2, x_3, x_4)$$

$$= \frac{1}{x_1 x_2 \partial x_4} ((x_3 - x_4) \varphi_0)(x_3, x_4) + \frac{1}{x_1 - x_2} \left\{ \frac{\partial}{\partial x_4} \left[ \left( 1 + \frac{x_3 - x_4}{x_1} \right) \Phi \right] (x_1, x_3, x_4) \right\}$$

$$\begin{aligned}
& -\frac{\partial}{\partial x_4} \left[ \left( 1 + \frac{x_3 - x_4}{x_2} \right) \Phi \right] (x_2, x_3, x_4) \Big\}, \\
F_2(x_1, x_2, x_3, x_4) \\
& = -\frac{1}{x_1 x_2} \frac{\partial}{\partial x_4} ((x_3 - x_4) \varphi_0)(x_3, x_4) + \frac{1}{x_1 - x_2} \left\{ \frac{\partial}{\partial x_4} \left[ \left( 1 - \frac{x_3 - x_4}{x_1} \right) \Phi \right] (x_1, x_3, x_4) \right. \\
& \quad \left. - \frac{\partial}{\partial x_4} \left[ \left( 1 - \frac{x_3 - x_4}{x_2} \right) \Phi \right] (x_2, x_3, x_4) \right\},
\end{aligned}$$

$$\begin{aligned}
F_3(x_1, x_2, x_3, x_4) \\
& = 2(\ln x_4 - 1) \varrho_{0,1}(x_1, x_2) - \frac{6(x_3 - x_4)}{x_1 x_2} - \frac{6(x_3 \ln x_3 - x_4 \ln x_4)}{x_1 x_2} \\
& \quad + \frac{x_1 x_2 + 2(x_1 + x_2)(x_3 - x_4)}{x_1^2 x_2^2} \varphi_0(x_3, x_4) - \frac{x_3 - 3x_4}{x_1 x_2} \frac{\partial \varphi_0}{\partial x_4}(x_3, x_4) \\
& \quad - \frac{x_4(x_3 - x_4)}{x_1 x_2} \frac{\partial^2 \varphi_0}{\partial x_4^2}(x_3, x_4) - \left( \frac{\partial}{\partial x_4} + x_4 \frac{\partial^2}{\partial x_4^2} \right) \Omega_0(x_1, x_2; x_3, x_4) \\
& \quad + \left( 1 - (x_3 - 3x_4) \frac{\partial}{\partial x_4} - x_4(x_3 - x_4) \frac{\partial^2}{\partial x_4^2} \right) \Omega_{-1}(x_1, x_2; x_3, x_4) \\
& \quad - \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right)^2 [\Omega_1(x_1, x_2; x_3, x_4) + (x_3 - x_4) \Omega_0(x_1, x_2; x_3, x_4)] \\
& \quad - 2 \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) \left[ \frac{\partial \Omega_1}{\partial x_4}(x_1, x_2; x_3, x_4) - (x_3 + x_4) \frac{\partial \Omega_0}{\partial x_4}(x_1, x_2; x_3, x_4) \right] \\
& \quad - 2(x_3 - x_4) \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) \Omega_{-1}(x_1, x_2; x_3, x_4),
\end{aligned}$$

$$\begin{aligned}
F_4(x_1, x_2, x_3, x_4) \\
& = 2(\ln x_4 - 1) \varrho_{0,1}(x_1, x_2) - \frac{6(x_3 - x_4)}{x_1 x_2} - \frac{6(x_3 \ln x_3 - x_4 \ln x_4)}{x_1 x_2} \\
& \quad - \frac{x_1 x_2 - 2(x_1 + x_2)(x_3 - x_4)}{x_1^2 x_2^2} \varphi_0(x_3, x_4) + \frac{x_3 + 3x_4}{x_1 x_2} \frac{\partial \varphi_0}{\partial x_4}(x_3, x_4) \\
& \quad - \frac{x_4(x_3 - x_4)}{x_1 x_2} \frac{\partial^2 \varphi_0}{\partial x_4^2}(x_3, x_4) + \left( -\frac{\partial}{\partial x_4} + x_4 \frac{\partial^2}{\partial x_4^2} \right) \Omega_0(x_1, x_2; x_3, x_4) \\
& \quad + \left( -1 + (x_3 + 3x_4) \frac{\partial}{\partial x_4} - x_4(x_3 - x_4) \frac{\partial^2}{\partial x_4^2} \right) \Omega_{-1}(x_1, x_2; x_3, x_4) \\
& \quad + \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right)^2 [\Omega_1(x_1, x_2; x_3, x_4) - (x_3 - x_4) \Omega_0(x_1, x_2; x_3, x_4)] \\
& \quad - 2 \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) \left[ \Omega_0(x_1, x_2; x_3, x_4) - 2x_4 \frac{\partial \Omega_0}{\partial x_4}(x_1, x_2; x_3, x_4) \right] \\
& \quad - 2(x_3 - x_4) \left( \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} \right) \Omega_{-1}(x_1, x_2; x_3, x_4),
\end{aligned}$$

$$\begin{aligned}
F_5(x_1, x_2, x_3, x_4) \\
& = -2(2 + \ln x_4) \varrho_{0,1}(x_1, x_2) + \frac{1}{x_1 x_2} \varphi_0(x_3, x_4) - \frac{x_3 - x_4}{x_1 x_2} \frac{\partial \varphi_0}{\partial x_4}(x_3, x_4)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\partial \Omega_0}{\partial x_4}(x_1, x_2; x_3, x_4) + \left(1 - (x_3 - x_4)\frac{\partial}{\partial x_4}\right)\Omega_{-1}(x_1, x_2; x_3, x_4), \\
& F_6(x_1, x_2, x_3, x_4) \\
& = 2(2 + \ln x_4)\varrho_{0,1}(x_1, x_2) - \frac{1}{x_1 x_2}\varphi_0(x_3, x_4) + \frac{x_3 - x_4}{x_1 x_2}\frac{\partial \varphi_0}{\partial x_4}(x_3, x_4) \\
& - \frac{\partial \Omega_0}{\partial x_4}(x_1, x_2; x_3, x_4) - \left(1 - (x_3 - x_4)\frac{\partial}{\partial x_4}\right)\Omega_{-1}(x_1, x_2; x_3, x_4). \tag{A.1}
\end{aligned}$$

The concrete expression of  $\Phi(x, y, z)$  can be found in [10,25]. In the limit  $z \ll x, y$ , we can expand  $\Phi(x, y, z)$  according  $z$  as

$$\begin{aligned}
\Phi(x, y, z) &= \varphi_0(x, y) + z\varphi_1(x, y) + \frac{z^2}{2!}\varphi_2(x, y) + \frac{z^3}{3!}\varphi_3(x, y) + \frac{z^4}{4!}\varphi_4(x, y) \\
&+ 2z(\ln z - 1)(1 + \varrho_{1,1}(x, y)) \\
&- 2z^2\left(\frac{\ln z}{2!} - \frac{3}{4}\right)\left(\frac{x+y}{(x-y)^2} + \frac{2xy}{(x-y)^3}\ln\frac{y}{x}\right) \\
&- \frac{2z^3}{(x-y)^2}\left(\frac{\ln z}{3!} - \frac{11}{36}\right)\left(1 + \frac{12xy}{(x-y)^2} + \frac{6xy(x+y)}{(x-y)^3}\ln\frac{y}{x}\right) \\
&- 2z^4\left(\frac{\ln z}{4!} - \frac{25}{288}\right)\left(\frac{2x^3 + 58x^2y + 58xy^2 + 2y^3}{(x-y)^6}\right. \\
&\left.+ \frac{24xy(x^2 + 3xy + y^2)}{(x-y)^7}\ln\frac{y}{x}\right) + \dots \tag{A.2}
\end{aligned}$$

with

$$\varphi_0(x, y) = \begin{cases} (x+y)\ln x \ln y + (x-y)\Theta(x, y), & x > y; \\ 2x \ln^2 x, & x = y; \\ (x+y)\ln x \ln y + (y-x)\Theta(y, x), & x < y, \end{cases} \tag{A.3}$$

$$\varphi_1(x, y) = \begin{cases} -\ln x \ln y - \frac{x+y}{x-y}\Theta(x, y), & x > y; \\ 4 - 2\ln x - \ln^2 x, & x = y; \\ -\ln x \ln y - \frac{x+y}{y-x}\Theta(y, x), & x < y, \end{cases} \tag{A.4}$$

$$\varphi_2(x, y) = \begin{cases} \frac{(2x^2+6xy)\ln x - (6xy+2y^2)\ln y}{(x-y)^3} - \frac{4xy}{(x-y)^3}\Theta(x, y), & x > y; \\ -\frac{5}{9x} + \frac{2}{3x}\ln x, & x = y; \\ \frac{(2x^2+6xy)\ln x - (6xy+2y^2)\ln y}{(x-y)^3} - \frac{4xy}{(y-x)^3}\Theta(y, x), & x < y, \end{cases} \tag{A.5}$$

$$\varphi_3(x, y) = \begin{cases} -\frac{12xy(x+y)}{(x-y)^5}\Theta(x, y) - \frac{2(x^2+xy+y^2)}{(x-y)^4} \\ + \frac{2(x^3+14x^2y+11xy^2)\ln x - 2(y^3+14xy^2+11x^2y)\ln y}{(x-y)^5}, & x > y; \\ -\frac{53}{150x^2} + \frac{1}{5x^2}\ln x, & x = y; \\ -\frac{12xy(x+y)}{(y-x)^5}\Theta(y, x) - \frac{2(x^2+xy+y^2)}{(x-y)^4} \\ + \frac{2(x^3+14x^2y+11xy^2)\ln x - 2(y^3+14xy^2+11x^2y)\ln y}{(x-y)^5}, & x < y, \end{cases} \tag{A.6}$$

$$\varphi_4(x, y) = \begin{cases} -\frac{48xy(x^2+3xy+y^2)}{(x-y)^7} \Theta(x, y) - \frac{2(3x^3+61x^2y+61xy^2+3y^3)}{(x-y)^6} \\ + \frac{4(x^4+3x^3y-45x^2y^2-25xy^3) \ln x - 4(y^4+3y^3x-45x^2y^2-25yx^3) \ln y}{(x-y)^7}, & x > y; \\ -\frac{598}{2205x^3} + \frac{1}{210x^3} \ln x, & x = y; \\ -\frac{48xy(x^2+3xy+y^2)}{(x-y)^7} \Theta(y, x) - \frac{2(3x^3+61x^2y+61xy^2+3y^3)}{(x-y)^6} \\ + \frac{4(x^4+3x^3y-45x^2y^2-25xy^3) \ln x - 4(y^4+3y^3x-45x^2y^2-25yx^3) \ln y}{(x-y)^7}, & x < y. \end{cases} \quad (\text{A.7})$$

Here, the function  $\Theta(x, y)$  is defined as

$$\Theta(x, y) = \ln x \ln \frac{y}{x} - 2 \ln(x-y) \ln \frac{y}{x} - 2 \text{Li}_2\left(\frac{y}{x}\right) + \frac{\pi^2}{3}. \quad (\text{A.8})$$

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